Final Project Report

Award Number : DE-SC00019256

Project Title : A Global, Multi-Resolution Approach to Regional Ocean Modeling

Principal Investigator : Qiang Du, Verne Willaman Professor of Mathematics

Institute : Pennsylvania State University

Brief Project Summary

This is a collaborative research project between Pennsylvania State University, Colorado State University and Florida State University to develop multi-resolution algorithms for regional ocean modeling.

- Collaborators at other institutes]:

  Profssor Don Estep, Department of Mathematics Colorado State University.
  Professor Max Gunzburger, Francis Eppes Distinguished Professor of Scientific Computing & Director of the Department of Scientific Computing, Florida State University.

Technical Progress

We review some key results obtained under grant support.

- To address the multiscale nature of the time evolution for the regional climate models, we proposed to work on multirate time integration schemes. During the project, the PI worked with his post-doc Dr.Chabaud on the subject and they develop a hybrid implicit and explicit adaptive multirate time integration method to solve systems of time-dependent equations that present two significantly different scales. We adopt an iteration scheme to decouple the equations with different time scales. At each iteration, we use an implicit Galerkin method with a fast time-step to solve for the fast scale variables and an explicit method with a slow time-step to solve for the slow variables. We derive an error estimator using a posteriori analysis which controls both the iteration number and the adaptive time-step selection. We present several numerical examples demonstrating the efficiency of our scheme and conclude with a stability analysis for a model problem.

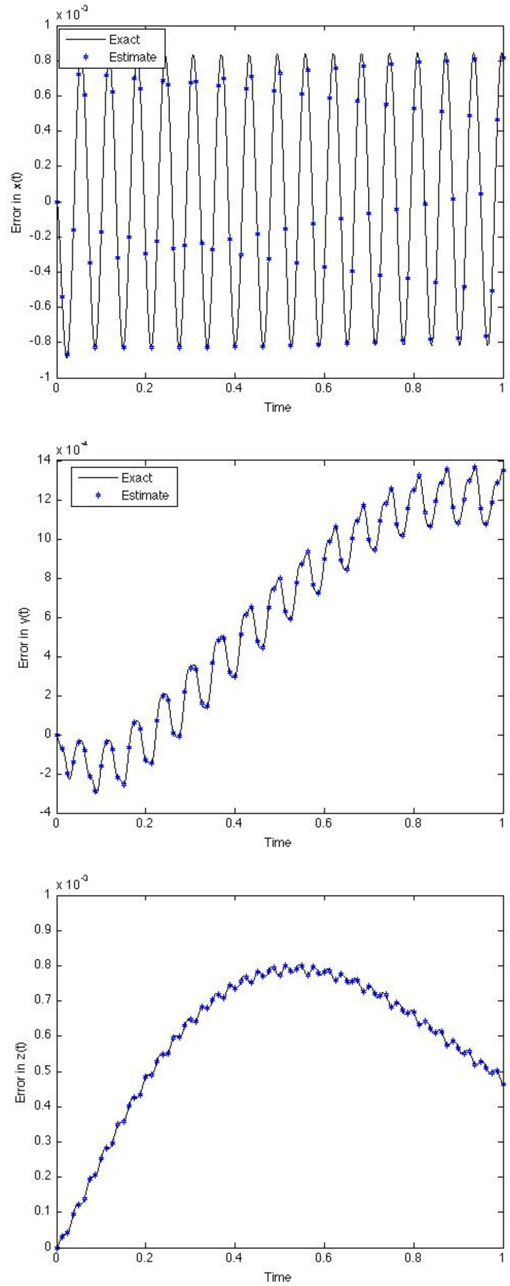

Figure 1: Comparison of actual and estimated errors for a 3D dynamic system [2].

- Efficient time integration of climate models depends on the choice of time step size which is determined by both accurate resolution and stability time-marching. It is known that spatial meshes might affect the choice of time-steps. Working with a former Ph.D student, we studied the effects of spatial simplicial meshes on the stability and the conditioning of fully discrete approximations of a parabolic equation using a general finite element discretization in space with explicit or implicit marching in time. Based on the new mesh dependent bounds on extreme eigenvalues of general finite element

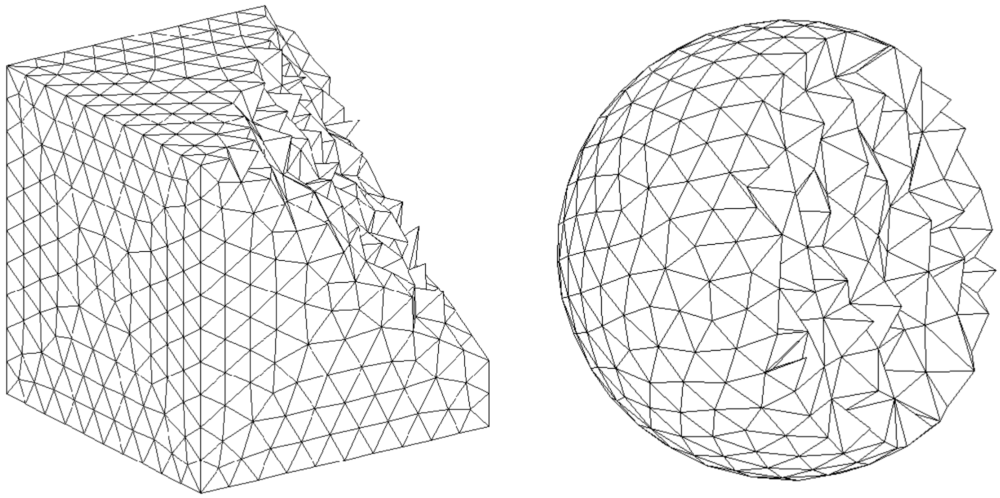

Figure 2: Cubic and spheric domains with CVDT meshes.

systems defined for simplicial meshes, we derive a new time step size condition for the explicit time integration schemes presented, which provides more precise dependence not only on mesh size but also on mesh shape. For the implicit time integration schemes, some explicit mesh-dependent estimates of the spectral condition number of the resulting linear systems are also established. Our results provide guidance to the studies of numerical stability for parabolic problems when using spatially unstructured adaptive and/or possibly anisotropic meshes.

- To increase spatial resolution for the numerical solution of PDEs, we considered a novel adaptive finite element method (AFEM) based upon the Centroidal Voronoi Tessellation (CVT) and superconvergent gradient recovery. Working with finite element solutions on high quality Centroidal Voronoi Delaunay Triangulation (CVDT) (see fig2), superconvergent recovery methods become particularly effective so that asymptotically exact a posteriori error estimations can be obtained. Through a seamless integration of these techniques, a convergent adaptive procedure is developed. As demonstrated by numerical examples, the new AFEM is capable of solving a variety of model problems and has great potential in practical applications including regional climate models.

Remaining balance of funds : $0.

Research personnels :

A couple of graduate students and post-doc scholars participated in the research effort. Notably, Dr. Brandon Chabaud, who got his Ph.D in applied mathematics from the University of Minnesota, has made important contributions. Upon completion of the project, he started working at the Los Alamos National Lab.

Deliverables: publications and presentations :

Publications:

- L. Zhu and Q. Du,
  Mesh-Dependent Stability for Finite Element Approximations of Parabolic Problems with Mass Lumping, J. Comp. Appl. Math., 236, 801-811, 2011.

- B. Chabaud and Q. Du,
  A hybrid implicit-explicit adaptive multirate numerical scheme for time-dependent equations, J. Scientific Computing, 51, 135-157, 2012.

- L. Zhu and Q. Du,
  Mesh dependent stability and condition number estimates for finite element approximations of parabolic problems, , Mathematics of Computation, 83, 37-64, 2014

- Y. Huang, H. Qin, D. Wang and Q. Du,
  Convergent adaptive finite element method based on centroidal Voronoi tessellations and superconvergence, Comm. Comp. Phys. , 10, 339-370, 2011.

Presentations:

- July 2010, SIAM Annual Meeting, Pittsburgh.

- Jan 2011, Joint Mathematics Meetings, New Orleans.

- Feb 2011, SIAM Penn State Student Chapter seminar.