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Comment on T. Tokano, 2013: "Wind-induced Equatorial Bulge in Venus and Titan General Circulation Models: Implications for the Simulation of Superrotation"

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Comment on T. Tokano, 2013: "Wind-induced equatorial bulge in Venus and Titan general circulation models: Implications for the simulation of superrotation" (GRL 40, 4538-4543)

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According to Tokano, "Table 1 shows that the centrifugal force in Earth's stratosphere is 5 orders of magnitude smaller than g, while it is only 4 and 3 orders of magnitude smaller on Venus and Titan, respectively . . . Thus, the relative contribution of the centrifugal force to the vertical momentum balance is not negligible at all on Titan and less negligible on Venus than on Earth, and the force balance is not accurately enough represented in hydrostatic GCMs." But Tokano's analysis omits an important term in Earth's momentum balance. Inclusion of this term contradicts the implication that an atmospheric "equatorial bulge" is more important on Venus than on Earth (though he may be right that an atmospheric equatorial buldge is very important on Titan).

The analysis leading to Tokano's Table 1 is based on the following equations:

$$\frac{1}{\rho}\frac{\partial p}{\partial z} + g = \frac{u^2+v^2}{r} + 2\,\Omega\,u\cos(\phi) \tag{A6}$$

$$\frac{\partial \Phi}{\partial \sigma} + \frac{R}{\sigma}\left(T + \frac{u^2+v^2+2\,\Omega\,u\cos(\phi)\,r}{r\,g}\,T_0\right) = 0 \tag{A7}$$

$$\frac{\partial \Phi'}{\partial \sigma} = -\frac{R}{\sigma}\left(T' + \frac{u^2}{r\,g}\,T_0\right) \tag{1}$$

Eq. (A6) is the vertical component of momentum conservation written in spherical coordinates with the usual notation (*e.g.* Eq. (2.21) in Holton 2004). The acceleration and friction terms are omitted, but the centrifugal (or "metric") term and the Coriolis term are retained on the right-hand side. Eq. (A7) follows from Eq. (A6) after switching the vertical coordinate from altitude to $\sigma = p/p_{\text{surface}}$ and assuming that the fraction appearing on the right-hand side is small. If this fraction were exactly zero then Eq. (A7) would become the hydrostatic equation in σ coordinates (*e.g.* Eq. (10.35) in Holton 2004). Assuming that the fraction is nonzero but small produces a "quasi-hydrostatic" system of equations. Under this assumption one can write $T = T_0 + T'$, where the subscript denotes a global mean, and replace T with T_0 in Eq. (A7), neglecting the product of two relatively small quantities. So far so good.

To derive Eq. (1) from Eq. (A7) Tokano subtracts the global mean hydrostatic equation (which replaces Φ with Φ' and T with T') and then drops several terms from the fraction in Eq. (A7). Neglecting v^2 compared with u^2 is a reasonable approximation, as is neglecting the Coriolis term <u>for Venus and probably for Titan</u>. But neglecting the Coriolis term while retaining u^2 is not reasonable for Earth because

$$u^2 + 2\,\Omega\,u\cos(\phi)\,r = u\,(u + 2\,\Omega\,r\cos(\phi))$$

and for Earth $2\,\Omega\,r$ is nearly 1000 meters / second, so the second term vastly exceeds the first at nearly all latitudes.

The essential flaw in the reasoning is best seen in an inertial (non-rotating) frame of reference. In this frame of reference Earth's stratospheric zonal wind is 400-500 meters / second at the Equator. But for Venus, the solid planet's rotation is so small that the corresponding zonal wind is of order 100 meters / second in both the rotating and non-rotating frames. From this point of view, it is clear that the equatorial bulge effect is more important in Earth's atmosphere.

Table 1 should be revised accordingly, adding $2\,\Omega\,r\cos(\phi)$ to u^2 on the right-hand side of Eq. (1). This will make no difference at the poles ($\phi = 90\,°$) but will add up to $2\,\Omega\,r$ elsewhere. Thus an appropriate revision of the table would be to add a final row showing $\left(u^2 + 2\,\Omega\,r\right) T_0 / r\,g\,T'$. Including data for Ω:

In[1]:=
```
worldName = {"Earth", "Venus", "Titan"};
```

In[2]:=
```
Ω = 2 π / Table[Abs[AstronomicalData[worldName[[i]], "RotationPeriod"]] s, {i, 1, 3}]
```

Out[2]=
$$\left\{\frac{0.000072921150}{s}, \frac{2.99246\times10^{-7}}{s}, \frac{4.559\times10^{-6}}{s}\right\}$$

and data present in the original version of the table:

In[3]:=
```
u = {50, 140, 200} m s^-1;
```

In[4]:=
```
T0 = {260, 230, 170} K;
```

In[5]:=
```
Tprime = {20, 10, 20} K;
```

In[6]:=
```
r = {6421, 6122, 2875} km * 1000 m km^-1;
```

In[7]:=
```
g = {9.66, 8.67, 1.09} m s^-2;
```

The final row of the table is thus modified from

In[9]:=
$$\frac{u^2\,T0}{r\,g\,Tprime}$$

Out[9]=
```
{0.000523967, 0.0084932, 0.108496}
```

to

In[10]:=
$$\frac{\left(u^2 + 2\,\Omega\,r\,u\right)\,T0}{r\,g\,Tprime}$$

Out[10]=
```
{0.0103374, 0.00871548, 0.122718}
```

In[11]:=
$$percentDifference = \frac{\% - \%\%}{\%} \times 100$$

Out[11]=
```
{94.9313, 2.55037, 11.5891}
```

Inclusion of the Coriolis term increases the ratio only slightly for Venus and Titan, but it doubles the ratio for Earth, making it somewhat greater than for Venus. Thus our results are consistent with Tokano’s inference that the equatorial bulge effect is very important for Titan. We find, however, that if the effect is important for Venus, then it is even more important for Earth.