LA-UR-15-23878

Title: DREAM3D simulations of inner-belt dynamics

Author(s): Cunningham, Gregory Scott

Intended for: Presentations from the International Workshop on Energetic Particle Processes in Near-Earth Space are being collected for distribution to the attendees.

Issued: 2015-05-26

70 YEARS OF CREATING TOMORROW

Los Alamos
NATIONAL LABORATORY

DREAM3D simulations of inner-belt dynamics

GS Cunningham

Space Science and Applications Group, ISR-1, LANL

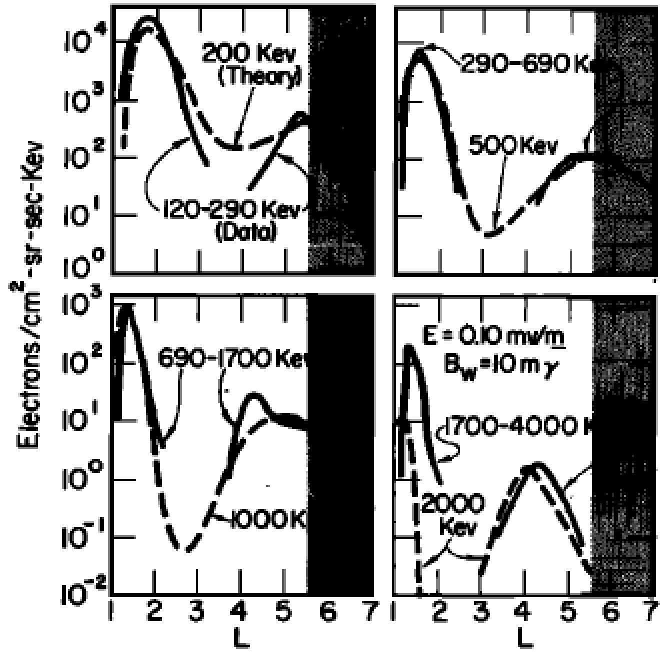
NNSA
National Nuclear Security Administration

Abstract

A 1973 paper by Lyons and Thorne explains the two-belt structure for electrons in the inner magnetosphere as a balance between inward radial diffusion and loss to the atmosphere, where the loss to the atmosphere is enabled by pitch-angle scattering from Coulomb and wave-particle interactions. In the 1973 paper, equilibrium solutions to a decoupled set of 1D radial diffusion equations, one for each value of the first invariant of motion, μ, were computed to produce the equilibrium two-belt structure. Each 1D radial diffusion equation incorporated an L- and μ-dependent `lifetime' due to the Coulomb and wave-particle interactions. This decoupling of the problem is appropriate under the assumption that radial diffusion is slow in comparison to pitch-angle scattering. However, for some values of μ and L the lifetime associated with pitch-angle scattering is comparable to the timescale associated with radial diffusion, suggesting that the true equilibrium solutions might reflect `coupled modes' involving pitch-angle scattering and radial diffusion and thus requiring a 3D diffusion model. In the work we show here, we have computed the equilibrium solutions using our 3D diffusion model, DREAM3D, that allows for such coupling. We find that the 3D equilibrium solutions are quite similar to the solutions shown in the 1973 paper when we use the same physical models for radial diffusion and pitch-angle scattering from hiss. However, we show that the equilibrium solutions are quite sensitive to various aspects of the physics model employed in the 1973 paper that can be improved, suggesting that additional work needs to be done to understand the two-belt structure.

Equilibrium two-belt structure explained with simple 1D diffusion model

$$\frac{\delta f(\mu,L,t)}{\delta t} = L^2 \frac{\delta}{\delta L}\left(\frac{D_{LL}(\mu,L,t)}{L^2}\frac{\delta f(\mu,L,t)}{\delta L}\right) - \frac{f(\mu,L,t)}{\tau(\mu,L,t)} = 0$$



From Lyons and Thorne, 1973

Lyons and Thorne assumed a ‘quiet-time’ inner magnetosphere

- Outer boundary at L=5.5 with flux given by
  - E>50 keV: Maxwellian with T=200 keV
  - E<50 keV: flux~KE$^{-0.75}$
- Dipole magnetic field
- Carpenter 1964 plasmasphere density, N=1000(4/L)^4
- ‘Electric potential’ radial diffusion only D^E_{LL}
  - Cornwall; equivalent to Brautigam&Albert with K_p=1
- Lifetime, τ
  - Coulomb collisions ($\tau \sim KE^{3/2}/N$)
  - 10 pT hiss at all MLAT, MLT, L (Gaussian in f, θ)
    - f_c=300 Hz, δf=300 Hz, f_{lc}=300 Hz, f_{uc}=? (2000 Hz)
    - $\tan(\theta_m)$=5

Lifetime is associated with an eigenvalue of the diffusion equation

- Suppose the solution to the pitch-angle diffusion equation $\frac{\delta f(\alpha,t)}{\delta t} = \frac{1}{T(\alpha)\sin(2\alpha)} \frac{\delta}{\delta\alpha}\left(D_{\alpha\alpha} T(\alpha)\sin(2\alpha) \frac{\delta f(\alpha,t)}{\delta\alpha} \right)$
- Can be written as $f(\alpha,t) = g(\alpha) e^{-t/\tau}$
- Then g(α) is an eigenmode of the diffusion equation

$$-\frac{1}{\tau} g(\alpha) = \frac{1}{T(\alpha)\sin(2\alpha)} \frac{\delta}{\delta\alpha}\left(D_{\alpha\alpha} T(\alpha)\sin(2\alpha) \frac{\delta g(\alpha)}{\delta\alpha} \right)$$

- The ‘slowest decaying mode’ has the largest τ, or the smallest eigenvalue

Lifetime is associated with an eigenvalue of the diffusion equation

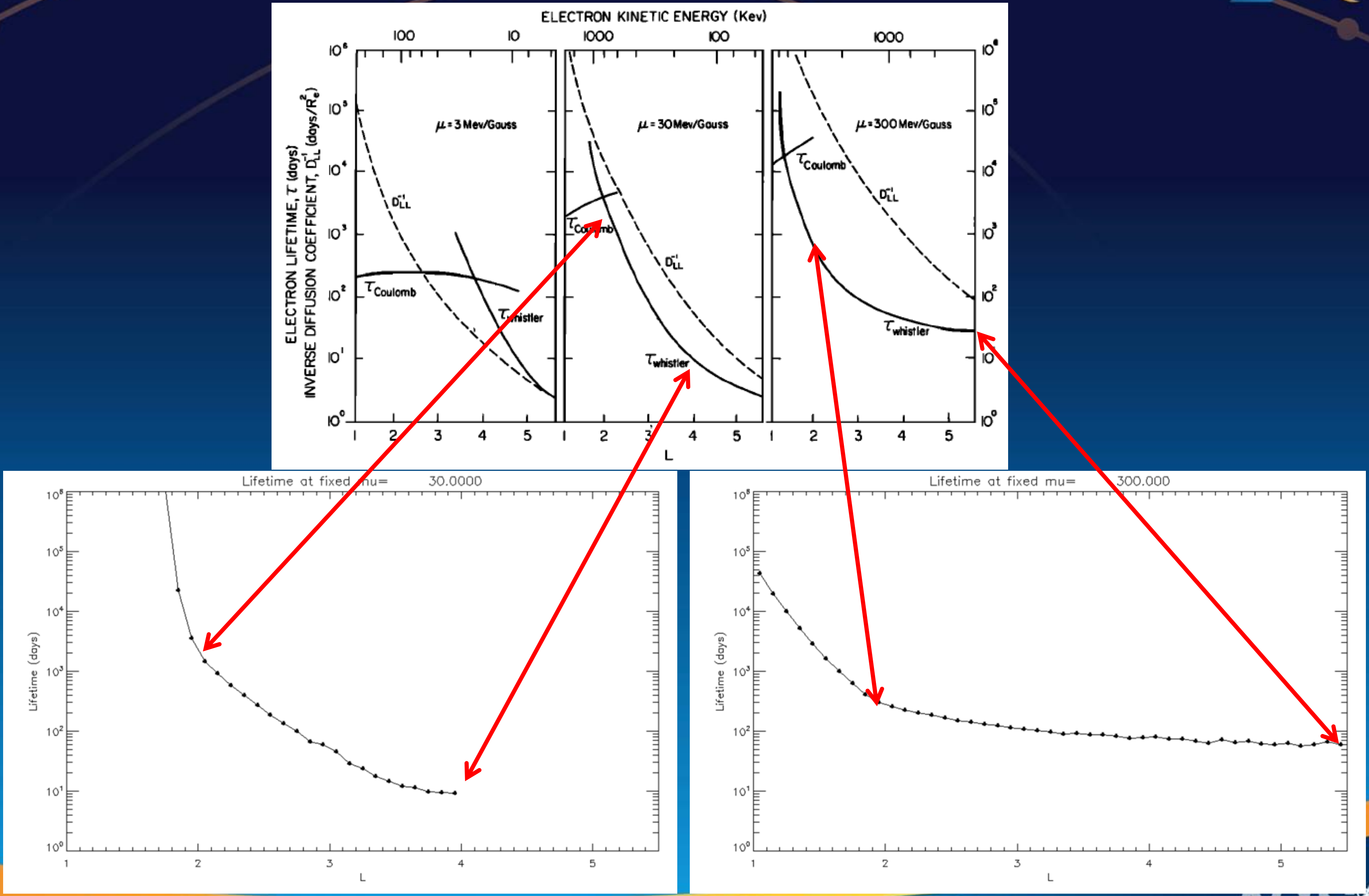
- Inversion of the diffusion equation (integrate both sides) gives an integral equation for which we want the largest eigenvalue

$$-\frac{1}{\tau}\int_{\alpha}^{\pi/2} T(\alpha')\sin(2\alpha')g(\alpha')d\alpha' = \left(D_{\alpha\alpha}T(\alpha)\sin(2\alpha)\frac{\delta g(\alpha)}{\delta\alpha}\right)_{\alpha}^{\pi/2} = D_{\alpha\alpha}T(\alpha)\sin(2\alpha)\frac{\delta g(\alpha)}{\delta\alpha}$$

$$-\int_{\alpha_{lc}}^{\alpha}\frac{1}{D_{\alpha\alpha}(\alpha'')T(\alpha'')\sin(2\alpha'')}\int_{\alpha''}^{\pi/2} T(\alpha')\sin(2\alpha')g(\alpha')d\alpha'd\alpha'' = g(\alpha)-g(\alpha_{lc}) = \tau g(\alpha)$$

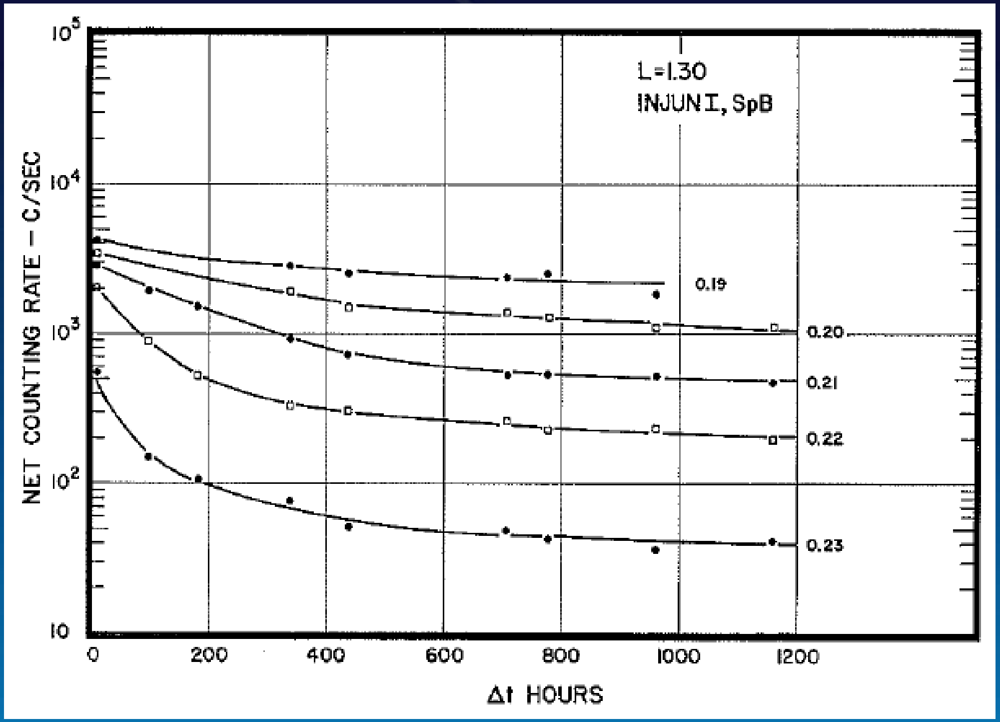
- Discretizing the distribution, g, produces a matrix problem, $Hg=\tau g$, which can be solved with repeated application of H to an initial guess. This is the ‘power method’ for finding the largest eigenvalue of a matrix.

DREAM3D lifetimes match L&T 73 well

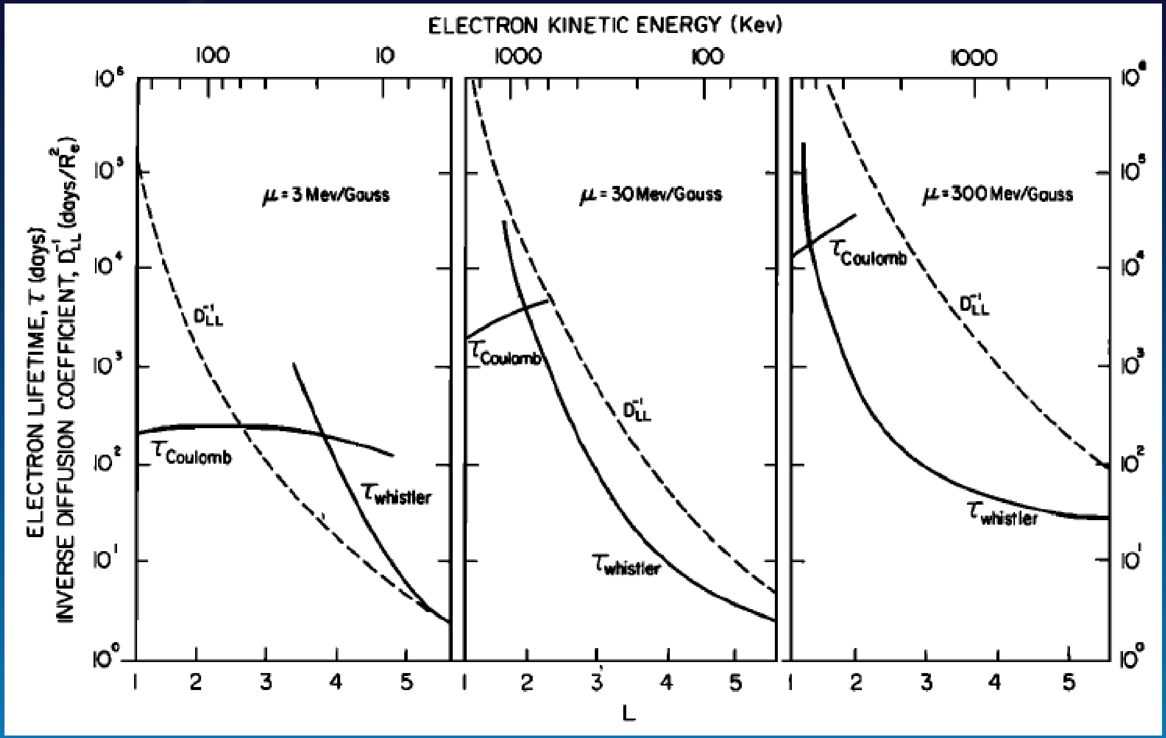


Slowest decaying mode does not always dominate the decay of fluxes

- Van Allen's msmts of artificial belt show changing decay rate due to change in pitch-angle distribution (Cowee)
- Schulz and Lanzerotti computed the ratio of the two smallest eigenvalues
- Injections to low L may complicate the picture (Reeves)



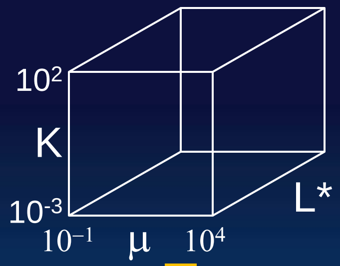
Possibility for coupling between radial diffusion and pitch-angle diffusion



When radial diffusion is slow (D^{-1}_{LL} is large) compared to loss from hiss (τ), then 1D approach is adequate.

Where the timescales for radial diffusion and loss are comparable, there is possibility of 'coupled' modes and need for 3D model.

DREAM3D is a 3D diffusion code implemented as 1D+2D diffusion



Initial condition based on Lyons and Thorne 1973 1D solutions

$$\frac{\delta f(\mu,K,L,t)}{\delta t} = L^2 \frac{\delta}{\delta L}\left(\frac{D_{LL}(\mu,L,t)}{L^2}\frac{\delta f(\mu,K,L,t)}{\delta L}\right) - \frac{f(\mu,K,L,t)}{\tau(\mu,L,t)}$$

$f(\mu_i,K_j,L_k,t_n)$

D I P O L E

$f(\mu_i,K_j,L_k,t_n+dt_1)$ ← D_{LL}, τ ← $K_p(t)=1$

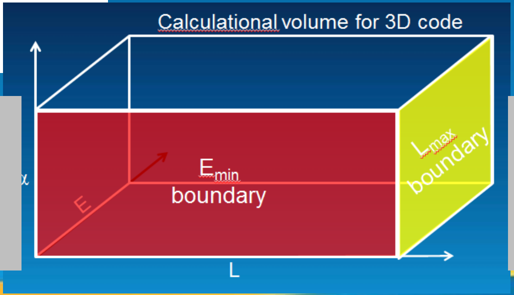
Coordinate conversion
(α_{eq},p,L) to (μ,K,L)

Static model of hiss → $D_{\alpha\alpha}$ → $F(\alpha_i,p_j,L_k,t_n+dt_1)$

$$\frac{\delta f(\alpha,p,L,t)}{\delta t} = \frac{1}{T(\alpha)\sin(2\alpha)}\frac{\delta}{\delta\alpha}\left(D_{\alpha\alpha}T(\alpha)\sin(2\alpha)\frac{\delta f(\alpha,p,L,t)}{\delta\alpha}\right) - \frac{f(\alpha,p,L,t)}{\tau(\alpha,L)}$$

$F(\alpha_i,p_j,L_k,t_n+dt_1)$

Coordinate conversion
(μ,K,L) to (α_{eq},p,L)



E_{min} boundary condition from Lyons and Thorne 1973 1D solution

L_{max} boundary condition same as Lyons and Thorne 1973 1D solution

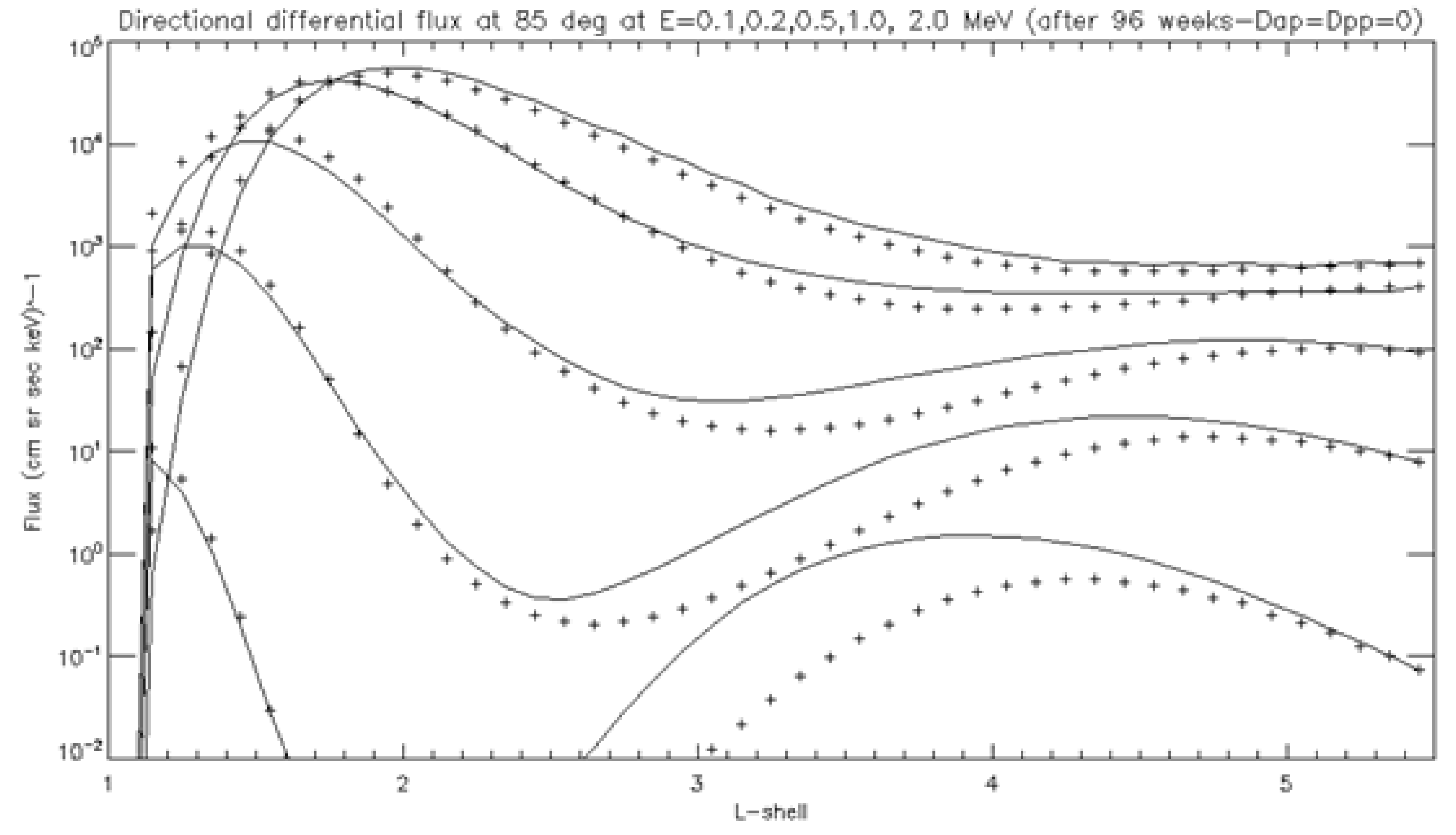
Numerical diffusion from conversion of (μ,K) to (α,p) is a concern

- Coordinate conversion occurs once per radial diffusion timestep, dt1 (0.5 hours in outer belt study, 1-56 days in this inner belt study)
- Assume (μ,K) grid is finer than (α,p) grid, so that binwidth in α is limitation. Using 180 bins in α

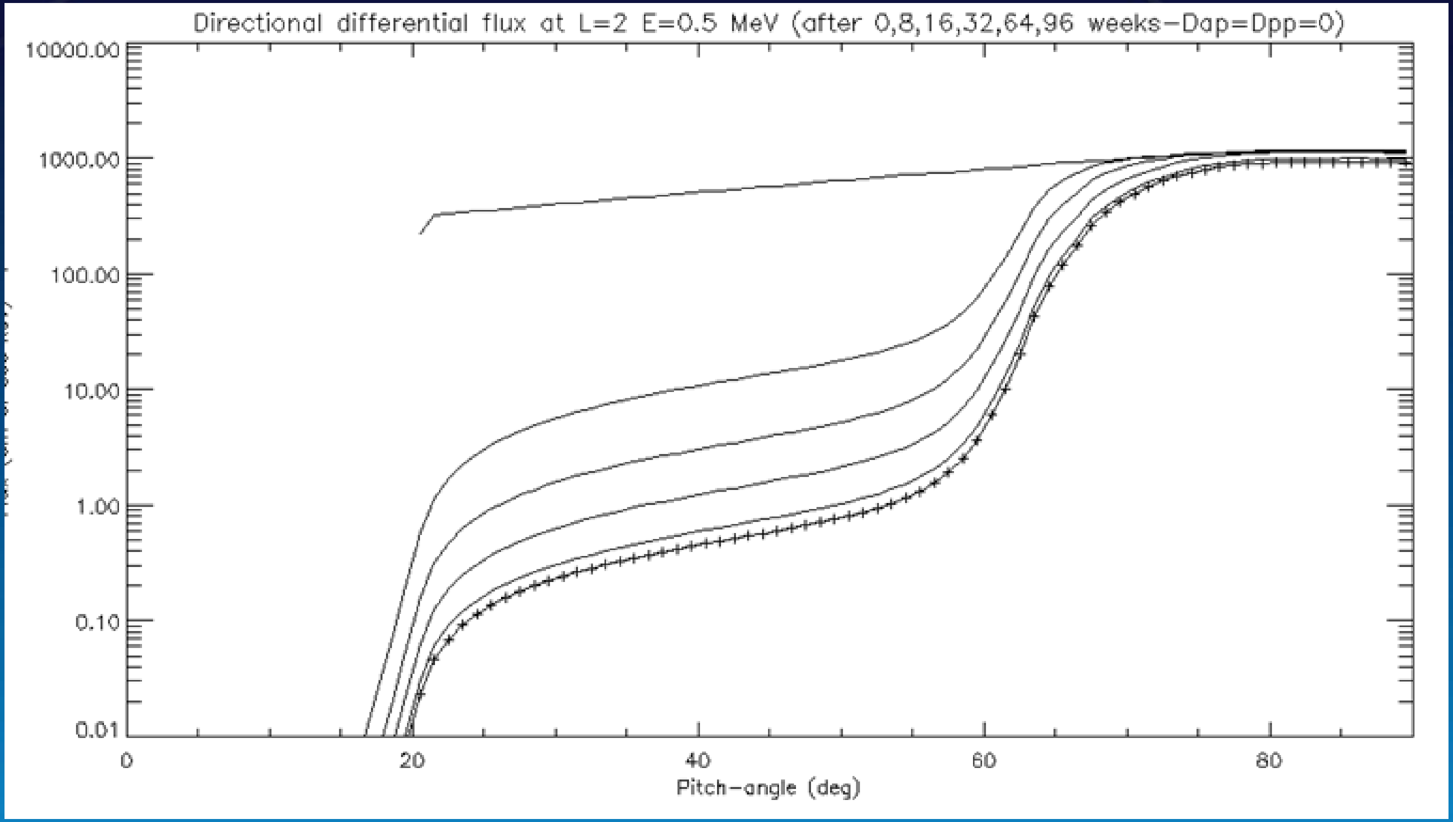
$$D_{\alpha\alpha}^{numerical} = \frac{\frac{1}{12}\left(\frac{\pi}{2\cdot 180}\right)^2}{24\cdot 3600} = 7.345\times 10^{-11} \text{ rads}^2\text{/sec}$$

  - yields a relatively small amount of numerical diffusion compared to other processes

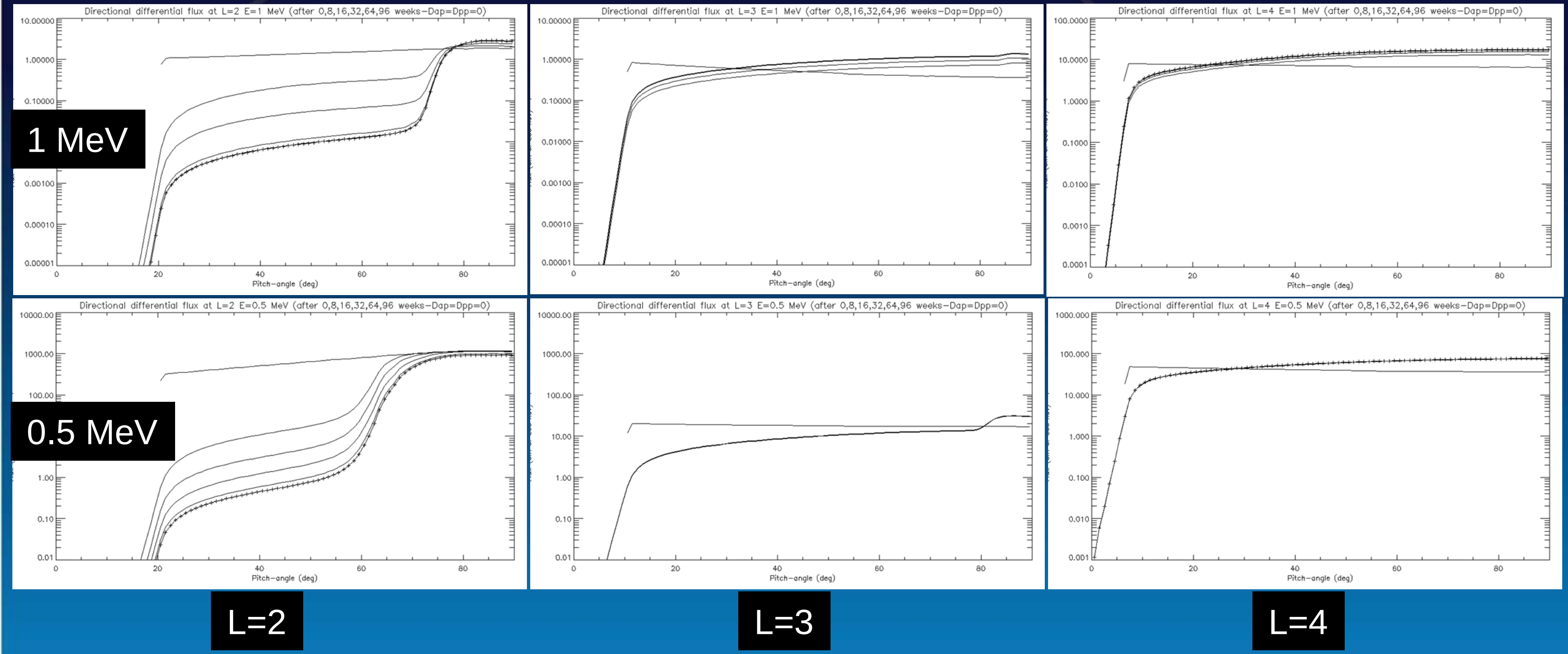
Results: some small amount of coupling at large energy and L



Convergence to equilibrium distribution can take years at high energy, low L



Top-hat feature in pitch-angle distribution appears at high-energy for low L-shell

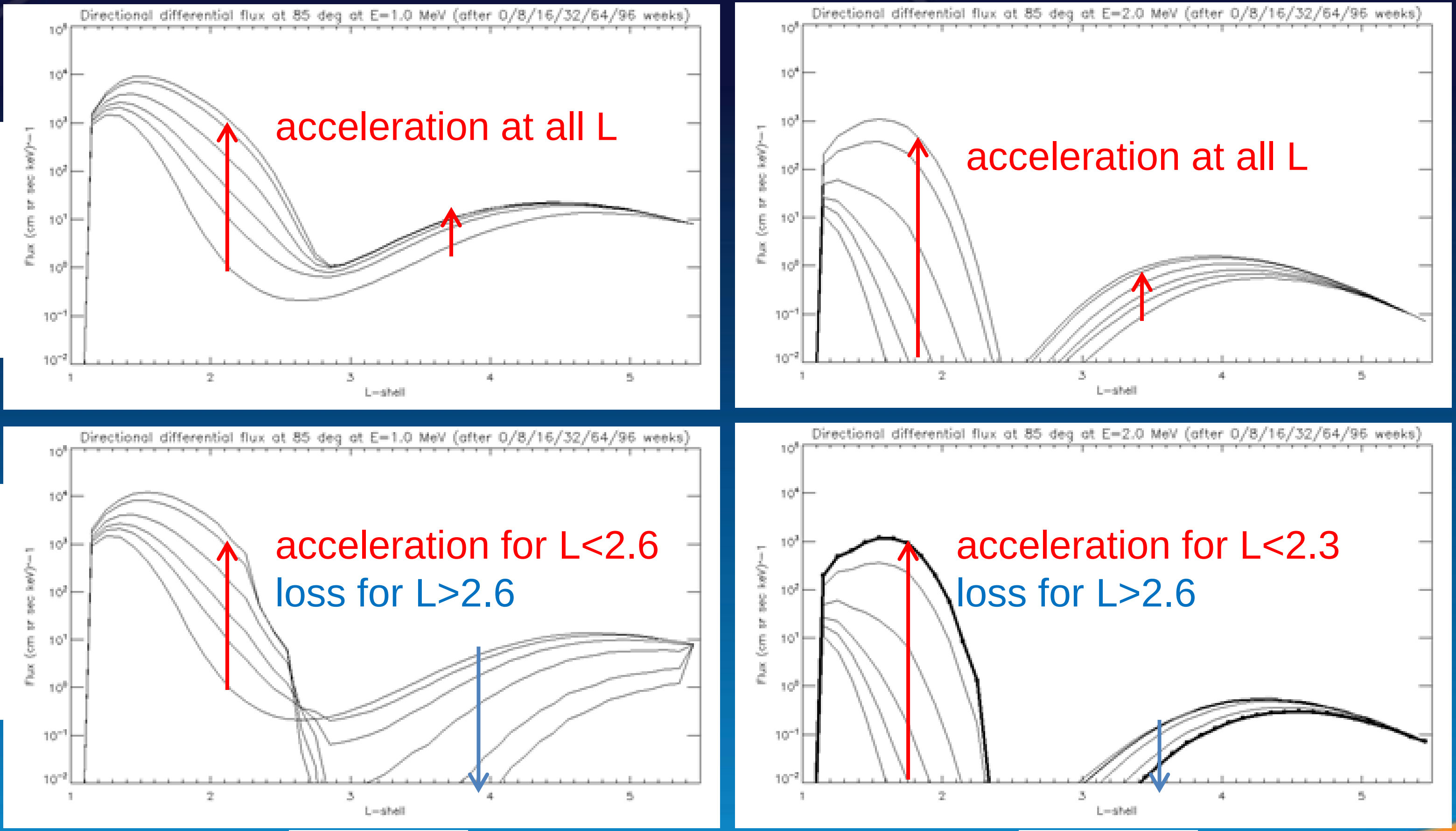


At low L, sharp gradient df/dα in pitch-angle distribution for all energies

Surprise! Acceleration caused by D_{ap} and large df/dα

Radial diffusion turned on dt1=1 day

Radial diffusion turned off dt1=56 days



Directional differential flux at 85 deg at E=1.0 MeV (after 0/8/16/32/64/96 weeks)

acceleration at all L

Directional differential flux at 85 deg at E=2.0 MeV (after 0/8/16/32/64/96 weeks)

acceleration at all L

Directional differential flux at 85 deg at E=1.0 MeV (after 0/8/16/32/64/96 weeks)

acceleration for L<2.6

loss for L>2.6

Directional differential flux at 85 deg at E=2.0 MeV (after 0/8/16/32/64/96 weeks)

acceleration for L<2.3

loss for L>2.6

1 MeV

2 MeV

Summary

- DREAM3D reproduces the Lyons and Thorne 1973 equilibrium two-belt structure using the same physical model
  - relatively minor coupling between radial and pitch-angle diffusion ‘modes’
  - a more modern and dynamic model would be interesting to implement
- Convergence to equilibrium state takes years at high energies and low L; what about injections?
- Surprise! Mixed pitch-angle momentum diffusion and large gradients in pitch-angle distribution cause acceleration in the model and big changes to equilibrium distribution

Modern inner magnetosphere models are significantly different from L&T 1973

- Carpenter 1992 plasmasphere density $N=10^{-0.3145L+3.9043}$
  - >10x smaller than Carpenter 1964 for L<=1.9
- Plasmapause location is dynamic
  - $L_{pp}=5.6-0.46K_p \sim 5.1$ for $K_p=1$
- Radial diffusion is dynamic (Brautigam and Albert 2005 JGR)
- Hiss amplitude is activity-dependent, position-dependent and broader in frequency (Li et al 2015 JGR)
- Higher-frequency VLF waves are important at lower L (Abel and Thorne 1998 JGR)
  - Lightning-generated whistler database (Meredith 2007 JGR)
  - Ground-based transmitters (Starks 2008 JGR, Cohen 2012 GRL)