Modeling Uncertainty in Coal Resource Assessments, With an Application to a Central Area of the Gillette Coal Field, Wyoming

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U.S. Department of the Interior
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Cover: View of a section of complex coal stratigraphy in an exposed mine highwall, Powder River Basin, Campbell County, Wyoming, USA. Photograph by James Luppens, USGS.

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By Ricardo A. Olea and James A. Luppens

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U.S. Geological Survey
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Conversion Factors

English to SI

Multiply	By	To obtain
	Length	
foot (ft)	0.3048	meter (m)
mile (mi)	1.609	kilometer (km)
	Area	
acre	4,047	square meter (m^2)
	Volume	
acre-foot (acre-ft)	1,233	cubic meter (m^3)
	Mass	
ton, short (2,000 pounds)	0.9072	metric ton (t)
	Density	
short ton/acre-foot	0.0007306	metric ton per cubic meter (t/m^3)

SI to English

Multiply	By	To obtain
	Length	
meter (m)	3.281	foot (ft)
kilometer (km)	0.6214	mile (mi)
	Area	
square meter (m^2)	0.0002471	acre
	Volume	
cubic meter (m^3)	0.0008107	acre-foot (acre-ft)
	Mass	
metric ton (t)	1.102	ton, short (2,000 pounds)
	Density	
metric ton per cubic meter (t/m^3)	1,368.778	short ton/acre-ft

Vertical coordinate information is referenced to North American Vertical Datum of 1988 (NAVD 88).

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Abstract

Standards for the public disclosure of mineral resources and reserves do not require the use of any specific methodology when it comes to estimating the reliability of the resources. Unbeknownst to most intended recipients of resource appraisals, such freedom commonly results in subjective opinions or estimations based on suboptimal approaches, such as use of distance methods. This report presents the results of a study of the third of three coal deposits in which drilling density has been increased one order of magnitude in three stages. Applying geostatistical simulation, the densest dataset was used to check the results obtained by modeling the sparser drillings. We have come up with two summary displays of results based on the same simulations, which individually and combined provide a better assessment of uncertainty than traditional qualitative resource classifications: (a) a display of cell 90 percent confidence interval versus cumulative cell tonnage, and (b) a histogram of total resources. The first graph allows classification of data into any number of bins with dividers to be decided by the assessor on the basis of a discriminating variable that is statistically accepted as a measure of uncertainty, thereby improving the quality and flexibility of the modeling. The second display expands the scope of the modeling by providing a quantitative measure of uncertainty for total tonnage, which is a fundamental concern for stockholders, geologists, and decision makers. Our approach allows us to correctly model uncertainty issues not possible to predict with distance methods, such as (a) different levels of uncertainty for individual beds with the same pattern and density of drill holes, (b) different local degrees of reduction of uncertainty with drilling densification reflecting fluctuation in the complexity of the geology, (c) average reduction in uncertainty at a disproportionately lesser rate than the reduction in area per drill hole, (d) the proportional effect of higher uncertainty in areas of higher tonnages, despite a regular drilling pattern, (e) the possibility of a local increase in uncertainty despite drilling densification to reflect a more complex geology as the deposit is known in more detail, and (f) for exactly the same drilling pattern, tonnage per individual beds with different uncertainty than the aggregated tonnage. These results should be considered realistic improvements over distance methods used for quantitative classification of uncertainty in coal resource, such as U.S. Geological Survey Circular 891.[1] The approach should be a welcome addition to the toolkit of Competent Persons preparing public disclosures according to international mineral codes such as those promoted by the Combined Reserves International Reporting Standards Committee (CRIRSCO)[2] and the Joint Ore Reserve Committee (JORC).[3]

Introduction

Mining ventures are risky and expensive endeavors. During recent decades, countries with important mining activities have prepared multiple standards to inform prospective investors and keep speculators away. Current globalization of markets and expansion in the operations of multinational companies have resulted in unification efforts to have better and universal standards, a trend that has been followed by better communications among ad hoc commissions (Njowa and others, 2014).

The largest organization promoting worldwide standards is the Combined Reserves International Reporting Standards Committee (CRIRSCO, 2013), established in South Africa in 1994 and also known as the Committee for Mineral Reserves International Reporting Standards. All main national and regional mining societies are members of CRIRSCO: Society

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[1] Wood, G.H., Jr., Kehn, T.M., Carter, M.D., and Culbertson, W.C., 1983, Coal resources classification system of the U.S. Geological Survey: U.S. Geological Survey Circular 891, 65 p.

[2] CRIRSCO (Combined Reserves International Reporting Standards Committee), 2013, International reporting template for the public reporting of exploration results, mineral resources and mineral reserves: Accessed February 2014 at *http://www.crirsco.com/crirsco_template_may2013.pdf*.

[3] JORC (Joint Ore Reserves Committee), 2012, Australasian code for reporting of exploration results, mineral resources and ore reserves: Accessed September 2014 at *http://www.jorc.org/docs/jorc_code2012.pdf*.

of Mining Engineers (SME), United States; Joint Ore Reserve Committee (JORC), Australasia; Canadian Institute of Mining, Metallurgy and Petroleum (CIM), Canada; Program Evaluation Resource Center (PERC), Europe; and South African Mineral Resource Committee (SAMREC), South Africa. The Australasian representation has been the most active and dominant group, having released two versions of its standards in the last 10 years (JORC, 2004, 2012). A parallel International Mineral Valuation Committee was formed in 2012 with the intention to accelerate the acceptance of consistent codes internationally.

Similar to efforts to abandon national laws in favor of an international order, development and adoption of international mineral standards has been difficult. So far, acceptance of past and present codes from committees such as CRIRSCO remains voluntary, accepted at the national society level rather than by regulatory agencies or courts of law. By far, countries continue to follow national regulations. The United States offers a typical example. Whereas the SME supports the CRIRSCO codes that are used internally by several mining companies, public disclosures must be prepared in accordance with significantly different regulations enforced by the U.S. Securities and Exchange Commission (SEC), which understandably relate primarily to economically mineable reserves (Rendu, 2006; SEC [n.d.]). For assessment of coal resources, the topic of our interest, the U.S. Geological Survey (USGS) follows yet a third code for disclosing uncertainty in the studies: U.S. Geological Survey Circular 891 (Wood and others, 1983).

Despite the disagreements, there has been significant progress advancing a world mining code, particularly in terms of requiring better sampling, demanding more rigorous laboratory assays, and using precise definitions. The JORC code acknowledges the peculiar nature of coal deposits given coal's stratiform and more continuous occurrence than other minerals in veins or dispersed in rocks (JORC, 2012, codes 42–44). International standards have eliminated the category of "hypothetical resources," maintaining only the other three traditional classes, which, in decreasing order of certainty, are "measured," "indicated," and "inferred." The weakest side of the codes is the failure to specify a methodology for calculating those three categories. All codes leave the selection of the methodology up to the authors of the reports, thus allowing inconsistencies, use of inadequate methods, and subjective opinions. Instead of researching methodologies, committees formulating mineral standards have devoted great efforts to specifying personal and ethical qualifications for an individual to be accepted as a "Competent Person" allowed to prepare a mining report. Thus, a clearly mathematical challenge has been resolved as an issue of individual character. The SEC does not even list minimal qualifications for Competent Persons.

In USGS Circular 891, the categories receive the names of measured, indicated, inferred and hypothetical, and are separated by dividers at ¼, ¾, and 3 miles (Wood and others, 1983). *Parsimony*[4] and simplicity are important considerations in mathematical modeling (Zellner and others, 2001). All other things being equal, a simpler method is better than a more complex one. It is difficult to improve on the parsimony in the simplest coal resources classification criteria; all that is needed in such methodologies is the distance to the closest drill hole(s). Many simple approaches accomplish their intended purpose as satisfactorily as more complex formulations; unfortunately, by and large, oversimplification has serious consequences (Rescher, 2007). It has been well demonstrated that distance is a poor surrogate for uncertainty in coal resource assessments for the simple reason that distance alone is not a significant determinant of the magnitude of errors (for example, Olea and others, 2011; Hohn and Britton, 2013). Adoption of distance alone as a surrogate for uncertainty is difficult to justify because there is wide consensus that several other variables have a significant impact on the accuracy of estimating resources in a coal deposit: (a) the geometric pattern of all drill holes, (b) the complexity of the coal bed boundaries, (c) the heterogeneity of the coal deposit, which is closely related to the depositional environment, (d) the degree of tectonic deformation, and (e) the number of coal beds. In addition, classifications based on distance do not provide any of the statistical information customarily reported in the risk analysis of other uncertain outcomes. Although mining codes do not even list the methods that should be used in the *estimations*, distance methods are a favorite classification tool of choice.

Geostatistics has been recognized for more than 60 years as offering satisfactory solutions to uncertainty problems in mining estimates (Krige, 1951; Matheron, 1963). As part of a continuous effort by the USGS to enhance and update scientific methods and practices, it was decided to evaluate the potential of geostatistics in the modeling of uncertainty in coal assessments as a replacement to the approach and classification system in USGS Circular 891. The current report is the last in a series of three studies to test the new methodology over a variety of coal deposits (table 1).

This study was a result of the encouraging, but expected, good results of the previous studies. The primary objective of this investigation was to test one last time the capability of geostatistical methods to model uncertainty, using a deposit from a different basin with markedly different geology. In this regard, the Gillette deposit discussed herein is a significantly larger deposit of better quality coal than either of the previously studied deposits and, consequently, has been of great economic interest to the mining industry. It is also important for the possibility of full disclosure because the data and locations are in the public domain.

Although the scope of this report is limited to the modeling of uncertainty in the assessment of coal resources, given the completely general character of geostatistics, the methodology applied here should give satisfactory results in the modeling of both resources and reserves of other mineral commodities, including oil and gas. The tools described and tested here should be a welcome addition to the methods to be considered by Competent Persons in the writing and disclosing of mining reports.

[4] The first occurrence of each term defined in the glossary appears in italics.

Table 1. Comparison of three studies by the U.S. Geological Survey to evaluate the merits of geostatistics for the modeling of uncertainty in coal resource assessments.

Site and study characteristics	Report		
	Olea and others (2011)	Olea & Luppens (2012)	This report
State	Texas	Louisiana	Wyoming
Nature of data	Confidential	Confidential	Public domain
Drilling	Three stages	Three stages	Progressive
Mining	No	No	Yes
Number of beds	One	Three	Four
Depositional environment	Deltaic	Deltaic	Fluvial
Coal quality	Lignite	Lignite	Subbituminous
Density, short ton/acre-foot	1,750	1,750	1,770
Study area	Entire deposit	Entire deposit	Central area only
Square cell side, feet	200	200	400
Depth of oxidation, feet	30, constant	Variable	35, constant
Resource mean value, billion short tons	0.14	0.58	39.0
Cell uncertainty proxy	Standard error	Standard error	5−95 spread
Reporting of results	Main findings only	Main findings only	Complete

Methodology

Early geostatistical methods were oriented toward generating a single estimate determined by minimizing mean square estimation errors, an approach generically known as *kriging*. Srivastava (2013) has prepared an overview of geostatistics applied to coal assessments. For attributes such as coal bed thickness, kriging results are summarized as two maps, one for the estimated values and the other for the *standard errors* of the estimated values. Over time, for applications in which the variability of estimates is the primary estimate, kriging has been superseded by *stochastic simulation*. Despite the still highly attractive property of minimizing estimation errors, kriging suffers from problems that are serious in many applications (Olea, 2009):

- The method produces smoothed versions of reality, with smoothing increasing as the data become more sparse; low values tend to be overestimated and high values tend to be underestimated.
- The regional properties of estimated values do not necessarily coincide with those of the data *sample*, let alone with those of the underlying *population* of true values; the *histograms* are different, and the spatial variability may be sufficiently different as to prevent good modeling (such as when poor reproduction of high-permeability streaks and impermeable barriers do not allow proper fluid flow modeling).
- In terms of modeling uncertainty, the two values for any location—estimate and standard error—are insufficient to define *confidence intervals*. To address this shortcoming, it is necessary to assume a *probability distribution* for the errors that is completely determined by the two parameters at hand—*mean* and *standard deviation*. By far the most common practice is to assume a *normal distribution*, which is not always a realistic choice.

Geostatistical stochastic simulation offers multiple methods for modeling reality on the basis of incomplete information (for example, Journel and Kyriakidis, 2004; Chilès and Delfiner, 2012). Common to all forms of modeling using stochastic simulation is that each simulation generates a different outcome—one of a theoretically infinite number of equally likely possibilities—and thus the outcome is a set of possible values rather than a unique answer. In the case of two-dimensional attributes such as geographical variation in bed thickness, results from geostatistical simulation can be displayed in the form of a set of maps, each of which has the same *probability* of being the error-free map. In the geostatistical terminology, each one of these outcomes is called a *realization*. In a way, these realizations are like those multiple maps typically obtained in response to requesting several experts to prepare, independently and manually, a contour map that reproduces data values at sampled locations (or, in geostatistical terminology, honoring the data). Variability in the results arises from the alternative possibilities of filling in missing values and yet honoring the data and the regional style of fluctuation expected for the attribute. By considering as many alternative outcomes as possible, the approach has some common element with creative thinking (de Bono, 1970). Properties of individual realizations differ from those of kriging in several ways: (a) Errors are no longer minimum in a mean square error sense, although, the *average* map of a large number of realizations converges toward a kriging map of estimates; (b) there is no smoothing; (c) regional properties of individual realizations, such as the histogram, match those of the data; and (d) rather than imposing specific distributions on estimation errors, the form of the distributions can be established directly from the set of simulated values.

To generate the simulated realizations, the study area is *tessellated* into square cells and values are simulated for each cell. Mapping of tessellations is done by graphically displaying grids of values at the center of each cell. Each realization usually comprises thousands of cells. The tessellation is kept the same for each realization. A detailed account of generating realizations can be found in the toolkit section at the back of this report, but in brief, realizations for each cell are generated by first simulating thicknesses and then multiplying these values by the appropriate coal density to generate tonnages. In the more general case, in which continuity of the deposit cannot be assumed, realizations of the presence or absence of the deposit (a binary indicator variable) are generated for each cell, and the thickness for each cell is multiplied by the corresponding indicator realization (0 for absence and 1 for presence).

Realizations contain implicit information about uncertainty. A simple visual inspection quickly provides a sense of the degree of similarity among maps. The more significant the differences are, the higher the uncertainty is. Visual inspection, however, is not the best way to assess uncertainty because there are at least two serious drawbacks: (a) a visual impression is not a quantitative evaluation, and (b) the appraisal is highly subjective.

There are more satisfactory approaches to extracting information about uncertainty from a collection of realizations. A highly successful statistical approach to modeling uncertainty is the concept of a *random variable*, which is at the core of stochastic simulation. In statistics, and by extension in geostatistics, when a quantity or event is known with certitude, it is equal to a single number. Two examples can illustrate the point:

1. It is not possible to know with certainty the outcome of rolling of a die; afterwards the result is unique, say, number 4.

2. Before exploiting a particular coal deposit, it is not possible to assign a number to the total tonnage, but after being completely mined out, the uncertainty vanishes and total tonnage is a single number, say, 7.1 million short tons.

When a quantity or event is unknown, statistics works with a list of all possible values together with their probability of occurrence. The collection of these values and associated probabilities of occurrence defines a random variable. In the case of a die, it is straightforward to list all possible outcomes of a roll: all integer numbers between 1 and 6. As for their associated probabilities, each face of a fair die is equally likely to occur, thus their probabilities are all equal to 1/6.

Uncertainty of Estimated Total Resources

Although the numerical modeling of the random variable that we shall call the ”total coal tonnage of a deposit” is not as straightforward as modeling the variable ”outcome of rolling a die”, the principle is the same. The use of multiple realizations is an increasingly acceptable approach, confirmed by the satisfactory results, from studies such as those listed in table 1. Considering that each realization is equally likely to provide, cell by cell, the tonnage in the deposit, summing up all cells in a realization provides one value of the total resource, in a manner analogous to throwing a die. The collection of values numerically defining a random variable is easier to analyze when displayed as shown in figure 1: figure 1*A* provides cumulative frequencies numerically modeling the *cumulative distribution* and 1*B* provides relative frequencies numerically modeling the probability distribution. In figure 1*A*, for example, there is a 95-percent probability that the true tonnage is less than 160 million short tons. It is not easy to read the proportion of points below any value in figure 1*B*, however, there is better appreciation of the rate of change in the slope of the cumulative function. Considering that only certain *percentiles* are customarily reported—5th, 25th, 50th (median), 75th, and 95th—the drawback of not being able to read percentiles is solved by listing those percentiles in the histogram. By calculating the difference between any two percentiles, it is possible to have statistical confidence intervals. Typically, only one of the graphs is reported. As it can be seen below, we will follow the more common practice of displaying the histogram.

Given a coal deposit or a study area, summaries such as those in figure 1 provide answers to the important question of the potential size of the resource. This way of reporting uncertainty in total resource estimates is standard in the assessment of oil and gas, in economics, and certainly in applied statistics. Consequently, a histogram of the uncertainty of total coal resource should require no further justification or explanation outside the coal assessment world. A histogram of uncertainty of the total estimated resource provides information impossible to extract from the coal resource classification of USGS Circular 891 (Wood and others, 1983). For example, for the case in figure 1, it is possible to state that there is a 90-percent probability that the deposit will have no less than 98.245 million short tons and no more than 159.585 million short tons, which can also be reported as a probability of 90 percent that the tonnage in the deposit is 128.915 ± 30.67 million short tons.

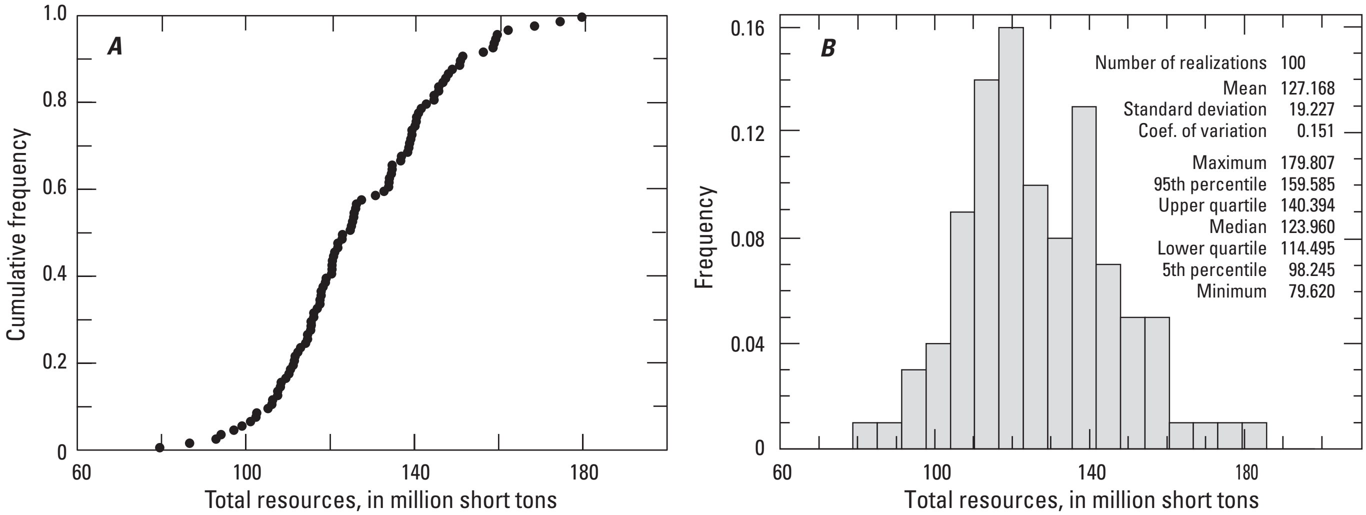

Figure 1. Uncertainty of total estimated resources in a study area. *A*, Cumulative frequencies of likely values. *B*, Histogram of relative frequencies for the same values. The glossary contains definitions for terms in the list of summary statistics.

Uncertainty of the Estimated Cell Tonnage

We have used a different processing of the same set of realizations to come up with an improvement of USGS Circular 891, thus providing a second measure of uncertainty offering a link between the old classification and the proposed approach. Preparation of the new measure is based on the same set of 100 realizations summarized in figure 1. This time, rather than summing the values of all cells in a realization, let us focus on the same cell in all the realizations, say, the one in the lower left corner of the study area. For each realization, that particular location may be inside or outside the boundary of the deposit. None of the cells outside the deposit contribute to the summary. In this particular case, there are 87 tonnage values only, corresponding to a proportion that has a precise meaning: there is an 87-percent probability that the example cell is inside the deposit. The 87 values are a numerical approximation to the random variable characterizing uncertainty in the tonnage at that cell in the lower left corner of the deposit (fig. 2).

The process is repeated for all the other cells, providing a random variable for all the thousands of cells tessellating the deposit, which are a numerical approximation to the *random function* behind the modeling. In this process we have significantly increased the amount of graphical information necessary to display by moving from 100 realizations to thousands of histograms. The ultimate interest this time is to link the total resource to the degree of uncertainty at the cell level. Instead of retaining all histograms, we can significantly reduce the amount of information by selecting one measure of uncertainty per histogram.

Uncertainty is related to the dispersion of the values, which can be measured, for example, by the standard deviation. Instead of retaining the entire histogram for each cell, we can retain only the standard deviation. In the sense that the set of simulated values for a given cell is representative of the complete range of possible values for that cell and the mean of these values is the expected value of the cell, the standard deviation is the standard deviation of an estimator, which is known as the standard error. For example, the standard error for the cell in the lower left corner of the study is 2.164 thousand short tons (fig. 2). The first improvement to the USGS 891 Circular would be to replace the discriminant variable used to make the classification. The notoriously underperforming “distance to closest drill hole” is replaced by the standard error. In addition, by eliminating the bins, the assessor is no longer restricted to a fixed number of

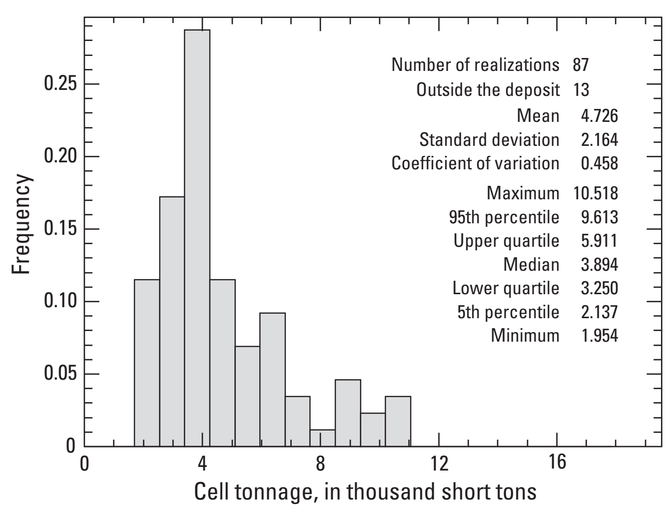

Figure 2. Model of tonnage uncertainty for a cell.

categories—measured, indicated, inferred and hypothetical in the case of USGS Circular 891—let alone constrained by fixed boundaries in the discriminating variable—¼ mile, ¾ mile, and 3 miles. Additional improvements can be gained by use of a cumulative type of display as shown in figure 3. The expected cell tonnage remains the attribute of interest. For a given cell, this value is obtained by determining the number of tonnage realizations and simultaneously considering the cells inside and outside the deposit according to all realizations. The expected cell tonnage is obtained as the mean of all cell values, counting as zero those cells outside the deposit. For example, for the cell in figure 2, the expected cell tonnage is equal to $(87 \cdot 4.726 + 13 \cdot 0)/100 = 4.112$, which is equivalent to the product of the mean times the probability that the cell is in the deposit: $0.87 \cdot 4.726$.

If the assessor prefers a classification in terms of a fixed number of bins, the graph in figure 3 is sufficiently flexible to provide information for preparing any bin classification of interest. The discriminating variable, however, should remain the truly discriminating standard error. Figure 4 shows an equivalent classification to USGS Circular 891 in the sense that it still considers four classes, but the boundaries are now the *quartiles*, namely, the *lower quartile*, the *median,* and the

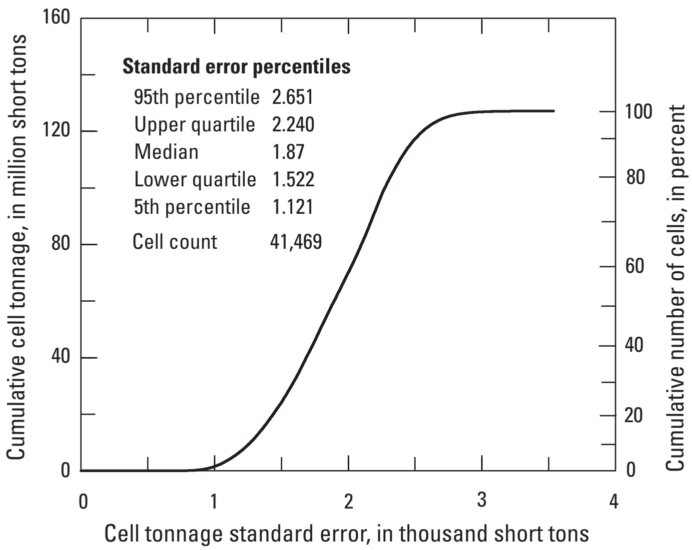

Figure 3. Contributions to the total resource as a function of cell uncertainty measured by the standard error.

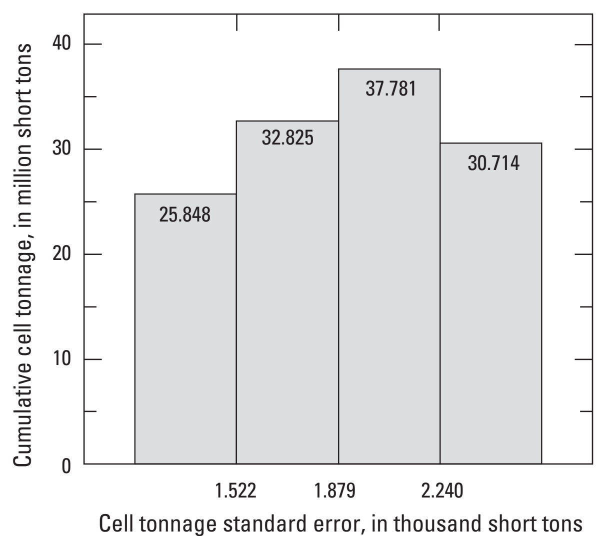

Figure 4. Classification of resources using as bins the standard error quartiles.

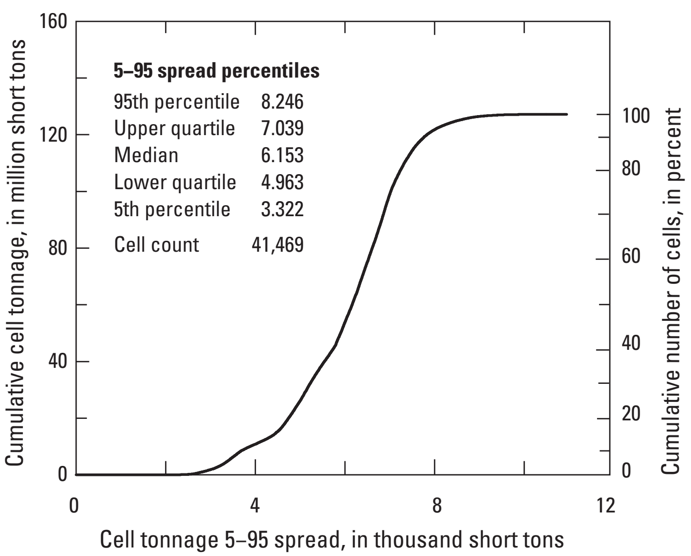

Figure 5. Contributions to total resources as a function of cell uncertainty measured by 5–95 spread.

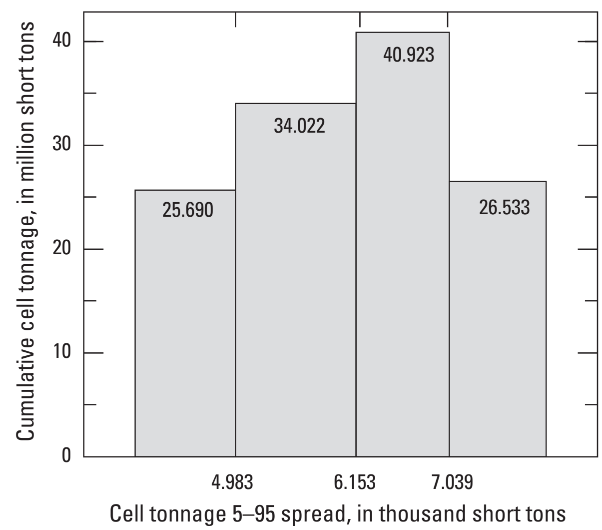

Figure 6. Classification of resources using as dividers the quartiles of the 5–95 spread.

upper quartile for the standard error. Certainly, there is no problem to reduce the number of categories to three if the intent is to follow the dictates of standards such as the JORC code.

Another measure of dispersion is any spread obtained as the difference between two percentiles. Practical interest is in pairs that are symmetric relative to the median, such as the 5th to 95th percentile spread (5–95 spread hereafter in this report), which is 7.476 for the histogram of values for the cell in figure 2. Figures 5 and 6 repeat the graphs in the previous illustrations for the case of the 5–95 spread. As for the case in figure 3, the second scale to the right of the graph allows the curve to be read as the cumulative frequency, in this case for the 5–95 spread. Consequently, for any percentage on the right-hand scale, the corresponding 5–95 spread value yields percentiles that can be tabulated, such as those to the right of the graph.

The results in terms of standard error and 5–95 spread are remarkably similar because they both model the uncertainty for the same data and, as expected, they are highly *correlated*. In this case, the correlation coefficient is 0.94; thus, one of the two uncertainty measures is highly redundant. Contrary to what was done in the previous studies (table 1), we decided to use the 5–95 spread because, compared to the standard error, it has the additional property of expressing uncertainty directly in terms of a confidence interval. By construction, the difference between the 5th and the 95th percentiles is the 90-percent confidence interval centered on the median. Thus, for example, an 8,000 short ton 5–95 spread is exactly twice as long as a 4,000 short ton interval; with the same 90 percent probability, the true value is likely to be within an interval twice as long, thus twice as uncertain. Only under special circumstances it is possible to calculate confidence intervals by using standard errors. One case is when all errors are assumed to follow a normal distribution, which, as confirmed by figure 2, is not usually the case for errors incurred in the estimation of mineral resources.

On the basis of experience gained with the experimental modeling of the deposits in table 1, we concluded that the most convenient way to summarize information about uncertainty in coal assessment, as provided by stochastic simulation, is via graphs such as those in figures 1*B* and 5. By construction, they sum the same cells in two different forms and thus, they give the same expected total tonnage. The tonnage for the sum of all cells in a display such as that in figure 5 is equal to the mean value in figure 1*B*. It is clear that it is not possible to prepare figure 5 starting from a graph such as figure 6, hence the superior power of a display in terms of cumulative tonnage. It is also clear that it is not possible to prepare figure 1 from either of figures 5 and 6. Instead, they provide complementary information.

The approach proposed in the report by no means exhausts the possibilities offered by geostatistics. For example, keeping the *sample size* constant, the spatial bootstrap provides a means of analyzing the sensitivity of the modeling to changes in the location of the drilled holes (Solow, 1985; Caumon and others, 2004). Because of the additional complexity required by such a modeling, it is recommended that the methodology be restricted to the level of sophistication used in this section.

Geology of the Study Area

The study area for this report is a subset of the Gillette coal assessment (fig. 7) within the Powder River Basin, Wyoming (Luppens and others, 2008). The Powder River Basin contains the largest deposits of low-sulfur, subbituminous coal in the world. In 2011, coal production from 16 mines in the basin totaled 462 million short tons—42 percent of the total coal production in the United States—making the Powder River Basin the single most important coal-producing basin in the Nation. About 426 million short tons (92 percent of total basin coal production) came from the Gillette coal field (Scott and Luppens, 2012).

From figure 7, it can be seen that the selection of the study area was based on both the availability of denser drilling data and the presence of several channels, which increase the geological complexity. Within the Gillette coal field, the strata dip between 1 and 2 degrees to the west. The four coal beds modeled in this area, from oldest to youngest, are the Canyon, Anderson, Smith, and Roland. All of the coal production to date in the Gillette coal field has come from the thick Canyon and Anderson coal beds, with maximum bed thicknesses reaching 138 and 134 feet, respectively. Throughout much of the modeled area, these two beds are separated by a relatively thin parting less than 10 feet in thickness. The coal bed geometry changes abruptly near Anderson channels that are penecontemporaneous with the coal beds (Luppens and others, 2008).

The north-south, strike-oriented cross-section (fig. 8) illustrates the influence of a relatively small east-west trending channel in T. 46 N., R. 73 W. (fig. 7). Southward, the Canyon and Anderson beds split and thin rapidly as they approach this channel. The Smith coal bed also thickens southward (fig. 8) where it reaches a maximum thickness of 85 feet in the southwest corner of the study area. Northward, the Anderson/Canyon parting also thickens and the Canyon bed thins significantly as it approaches a channel associated with Canyon bed deposition on the northern edge of the modeled area (fig. 7).

The east-west, dip-oriented cross section (fig. 9) illustrates the sequence of coal bed development. Current mining in the Gillette coal field in the Anderson/Canyon interval is east of the modeled area. As mining continues westward (deeper), the Smith bed will be encountered just east of the modeled area. Finally, the Roland bed whose subcrop lies in the eastern part of the modeled area will be recovered. The east-west cross section (fig. 9) also demonstrates the significant impacts of a major north-south trending channel in T. 47 N., R. 74 W. (fig. 7). The thickness of the Anderson/Canyon parting abruptly increases towards the channel and the Anderson bed actually pinches out.

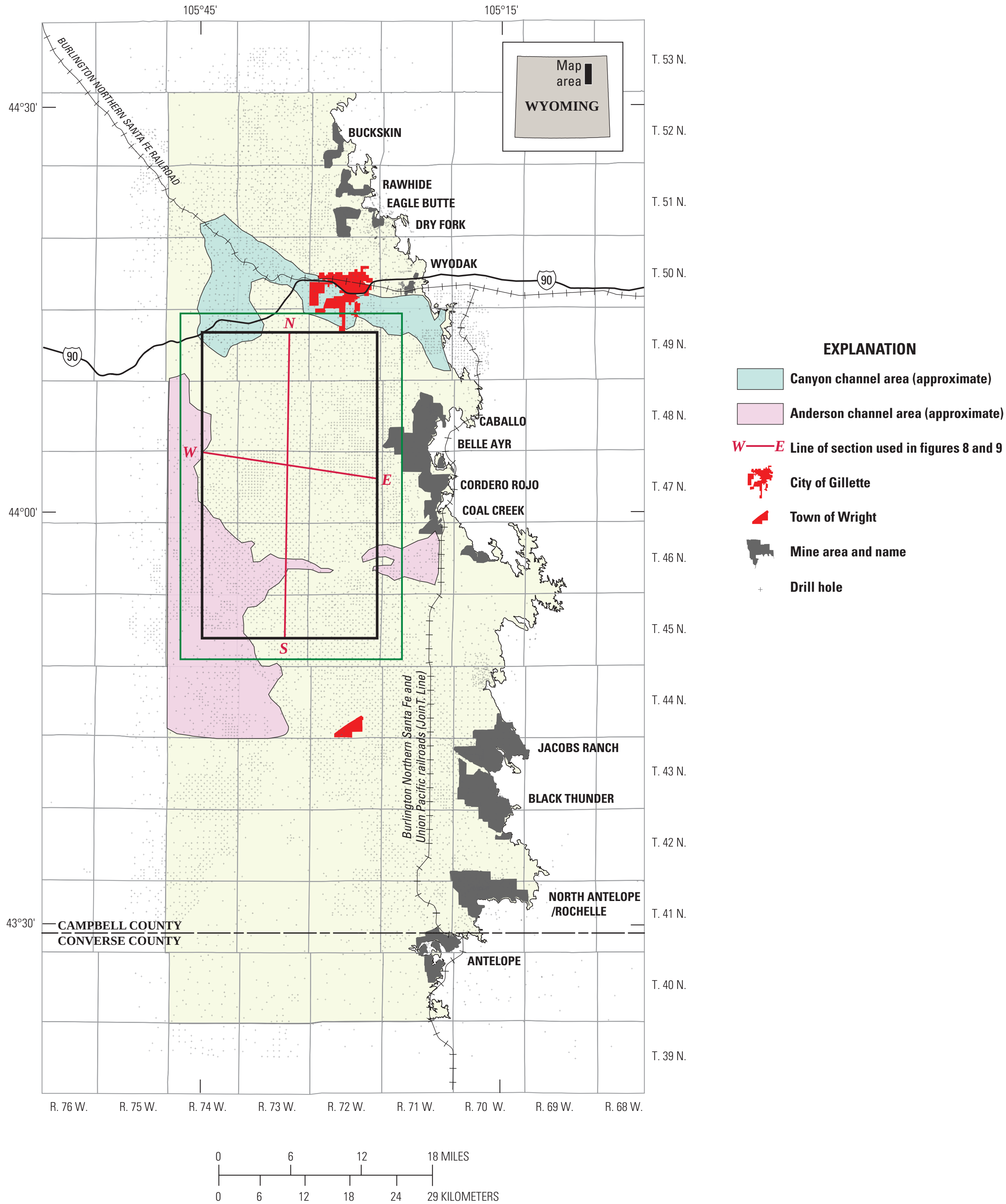

Figure 7. Location of study area. Inner rectangle denotes the actually simulated area, whereas the outer rectangle is a data capture area established to eliminate artificial boundary effects (modified from Luppens and others, 2008).

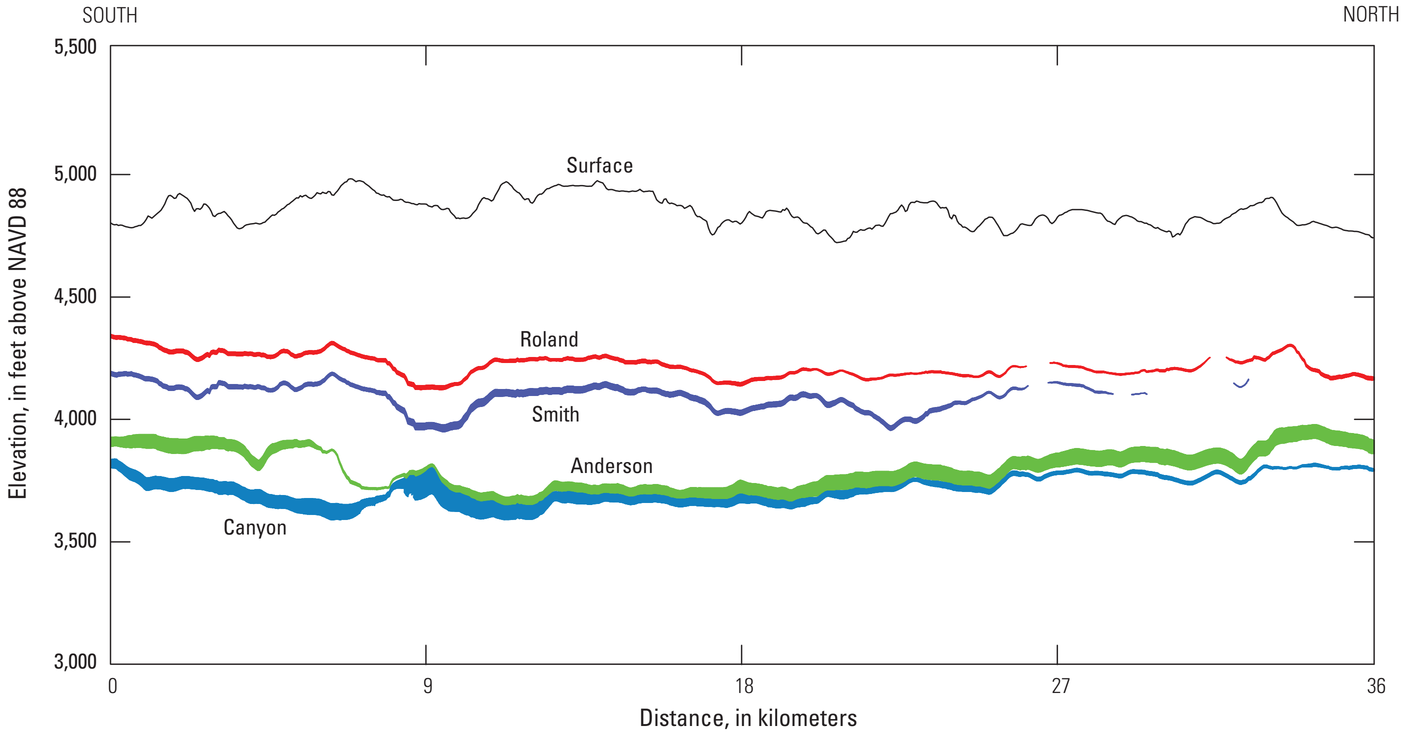

Figure 8. North-south cross section. Trace of section shown in figure 7.

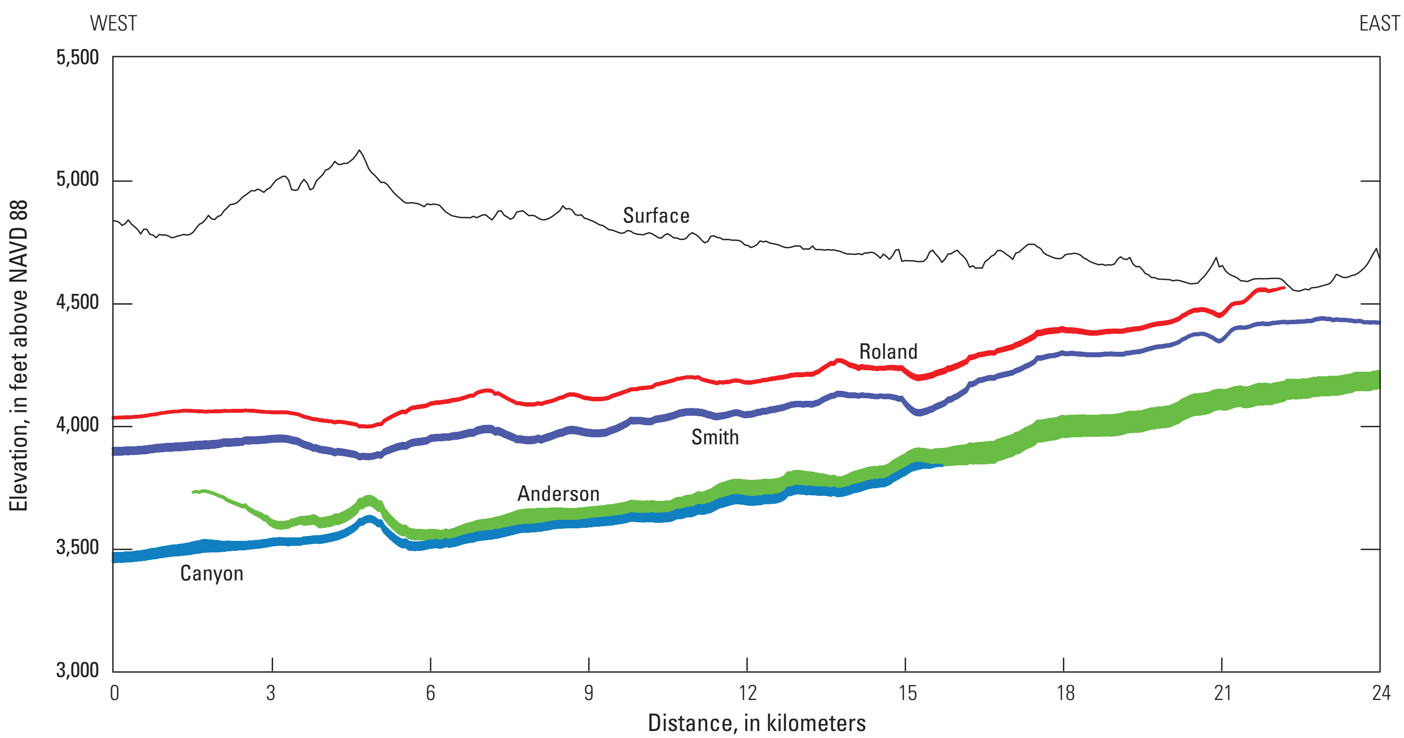

Figure 9. East-west cross section. Trace of section shown in figure 7.

Uncertainty Modeling of the Gillette Field

Data from previous studies (table 1) were collected in three successive stages—reconnaissance, infill, and development—as economic significance of the deposit was confirmed. Data collection in successive stages has the statistical advantage of allowing checking of results obtained by modeling the early drilling. Whilst there was no systematic drilling densification in the Gillette coal field, the concept and terminology were kept. Here, rather than having a true regular densification of data, we artificially replicated the idea, which, although historically inaccurate, is perfectly acceptable for the ultimate purpose of having information to validate results. All the drilling patterns tend to follow a regular square grid with gaps. Selected spacings are about 3 miles for the reconnaissance stage; approximately ¾ mile for the infill stage; and about ⅓ mile for development stage, except for a densification in township T. 47 N., R. 73 W. where it is ¼ mile. These spacings closely follow the boundaries in the USGS Circular 891 class boundaries. Given the quadratic relationship between spacing and area, the infill dataset has approximately 12 times the drill holes of the reconnaissance dataset and the development dataset has 45 times the drill holes of the reconnaissance dataset.

Intending to gain experience in the modeling of uncertainty instead of trying to reassess the entire coal field, we chose a fraction of the deposit for the study, but one that was large enough to present challenging geological anomalies for the characterization. We selected two areas, shown in figure 7. The inner study area was placed in the middle of a larger data capture area, large enough to eliminate any artificial boundary effects for ignoring data where indeed there were additional drill holes outside the study area. The study area was selected to focus the modeling on uncertainty issues related to geologic factors. Previous studies have demonstrated that the methodology is capable of properly handling the modeling of uncertainty related to true lack of data beyond the boundaries of a deposit.

Latitude and longitude for the data were converted to state plane coordinates for the purpose of generating the realizations. Township and range notation was used to annotate the final maps to be consistent with the most common practice in the Wyoming coal industry. Cell size was selected to be 400 feet, the size of the minimum details of interest to model. This is significantly below the 1,800-foot average distance to nearest drill hole in the densest dataset—the development drilling. The grid dimensions were 184 columns and 345 rows, for a total of 63,480 cells.

Thickness data were not aggregated at the drill-hole level. Instead, the modeling was purposely done bed by bed to maximize the number of opportunities to check the methodology. From oldest to youngest, the modeled coal beds discussed below are Canyon, Anderson, Smith, and Roland. The assumptions are that (a) the data cover the entire study area, (b) there are insufficient values for coal density to map them, and (c) there are no faults. Three coal beds were modeled by the use of Procedure I in the toolkit section (at back of report) because they are deep enough to have undergone significant weathering. The exception is the Roland coal bed, which was modeled by applying Procedure J in the toolkit.

Canyon Coal Bed

Thickness data for all three drilling stages at the Canyon coal bed are shown in figure 10. The same data are displayed in figure 11 after transformation to presence-absence thickness indicators and the first of a total of 100 realizations generated from the data available for the corresponding drilling stage. For example, no information from the development drilling stage was used to prepare the set of realizations of which figure 11*B* is a part. When a drill hole does not penetrate a coal bed after reaching sufficient depth, it is said that the coal bed is absent or missing.

Figure 12 shows tonnage realizations after combining the indicator and thickness realizations, and additionally converting thickness to tonnage using a factor of 1,770 short tons per acre-foot. The percentiles are based on the total tonnage per realization.

Figure 13 displays maps for numerical modeling of two significant cell statistics: mean and the spread between the 5th and 95th percentiles. It is interesting to note that these two statistics track each other fairly consistently: where the mean is low, the 5−95 spread tends to be low and, when the mean is high, the spread is high too. This pattern of variability is not uncommon in geology for attributes characterizing the magnitude of resources and the uncertainty in their modeling. In fact, it is common enough to have a special name: *proportional effect* (Manchuk and others, 2009). The cause is that variability of the resource is proportional to its magnitude. For example, suppose that, on average, variability of the tonnage around a cell is about 50 percent. Then, if there is a proportional effect, when tonnage is approximately 10 thousand short tons (kst), nearby cells will have tonnages as low as 5 kst or as high as 15 kst. If, in another more valuable part of the deposit, the tonnages are in the order of 100 kst, the variability will be 100 kst ± 50 kst rather than the

±5 kst seen for the lower tonnage area. This type of variability results in proportional means and measures of uncertainty, such as the 5–95 spread. It is typical of attributes that have positively skewed distributions and, more importantly, a realistic modeling of the magnitude of the errors and uncertainties encountered in the mining of deposits. This type of variability is particularly unfavorable for characterizing uncertainty in modeled resources in terms of distance to the nearest drill hole(s) because to a large extent the uncertainty is controlled by the geology, not by the drilling spacing.

Figure 14 displays summary statistics for the uncertainty. Several observations can be made about the data and results. Most anomalies in the modeling and subsequent results are consistent with the *Nyquist sampling theorem*, which states that it is not possible to reconstruct accurately an anomaly that is smaller than a critical distance of twice the sampling interval (Chaparro, 2011). For intervals closely above the critical distance, reproduction of reality by the drilling necessarily will be imperfect, an effect that is called *aliasing*. Evidently, details clearly below the critical distance will be completely missed. At the reconnaissance interval of 3 miles, anomalies smaller than 6 miles could be completely missed or reconstructed with aliasing. For the infill spacing of ¾ mile, the minimum size goes down to 1½ miles. The Nyquist frequency strictly applies to inverse distance and minimum mean square error interpolation procedures. Stochastic simulation does a better job predicting beyond the critical distance through the mechanism of inserting anomalies not fully supported by data. For example, in the case of the Canyon reconnaissance stage, figure 10 shows that the drilling missed the thickening of the coal bed at T. 46 N., R. 73 W. that constitutes the upper 5-percent tail of the infill and development distributions. However, all realizations in figure 12*A* through 12*C* provide enough information from the only 48 values of thickness to suggest a thick bed at T. 46 N., R. 73 W. in various degrees. The other 97 realizations not displayed here show similar tendencies. Because of the blurry and tentative nature of the anomaly in most of the 100 realizations, the anomaly is not as prominent when the realizations are averaged in figure 13*A* as is in the other two mean maps; however, it is distinctive enough to make clear to any mining company or prospecting agency that a local drilling densification is in order to clarify the aliasing.

Despite the aliasing, likely values for tonnage in figure 14*A* are comparable to those from the denser drilling. Although the true tonnage of the study area is unknown, drill-hole density in the development stage is high enough to be close to an exhaustive sample. Although the answer is not a single value, the outcome has been narrowed to the interval 9.4 to 9.9 billion short tons (bst), which is completely inside the range of possible values predicted with the other two, sparser drillings. As expected, the reconnaissance drilling has the likely values spread over the longest interval, denoting the highest uncertainty. Note that a 45-fold increase in the data from reconnaissance to development resulted in only a 4.6 times reduction in the standard deviation from figures 14*A* to 14*E*. Reduction of uncertainty by additional drilling is an expensive, nonlinear process.

When and adequate model is applied correctly, the results match reality. In the case of the 5–95 spread, 90 percent of cell tonnage values given by new drilling should be within intervals bounded by the 5th and the 95th percentiles defined by the realizations. Data provided by 1,349 new drill holes were used to check the reconnaissance modeling based on the 65 values in figure 10*A* and 10*B*. Exactly 90 percent of the new values fell inside the 5–95 spread, empirically confirming the degree of *coverage* implied by the interval.

The values in the abscissae of the graph in the right column of figure 14 are exactly those in the maps in the right column of figure 13. The curve in figure 14*F* shows a discontinuity at the origin related to all the blue dots in figure 13*F* that have no cell error because drilling has provided the exact values of thickness. As the number of drilled cells decreases in the other two graphs (figs 13*B* and 13*D*), the discontinuity is less noticeable; cumulative tonnage starts to be estimated with uncertainty at lower values. For example, the first 0.32 bst in figure 14*F* has been estimated with certainty. In figure 14*B*, the cell contributing to the 0.32 bst has a 5–95 spread of 150 kst. From there on, the discrepancy is less noticeable and eventually reverses. As seen later, this is an exception rather than the rule. Following the idea that uncertainty is related only to distance to the nearest drill hole, reconnaissance uncertainty should always be greater than infill and development uncertainty; it is, in this example, up to about 7 bst. Beyond that value, infill and development uncertainty are almost equally much higher. From figure 13, it can be seen that the higher 5–95 spread values are again associated with high thickness. In geology in general, and coal geology in particular, an increase in resolution resulting from an increase in the number and density of the data commonly reveals more intricate detail—with a consequent increase in variability—rather than simply confirming the variability on the scale of resolution previously available. An increase in the variability makes accurate predictions more difficult and, hence, increases uncertainty, leading to higher 5–95 spreads.

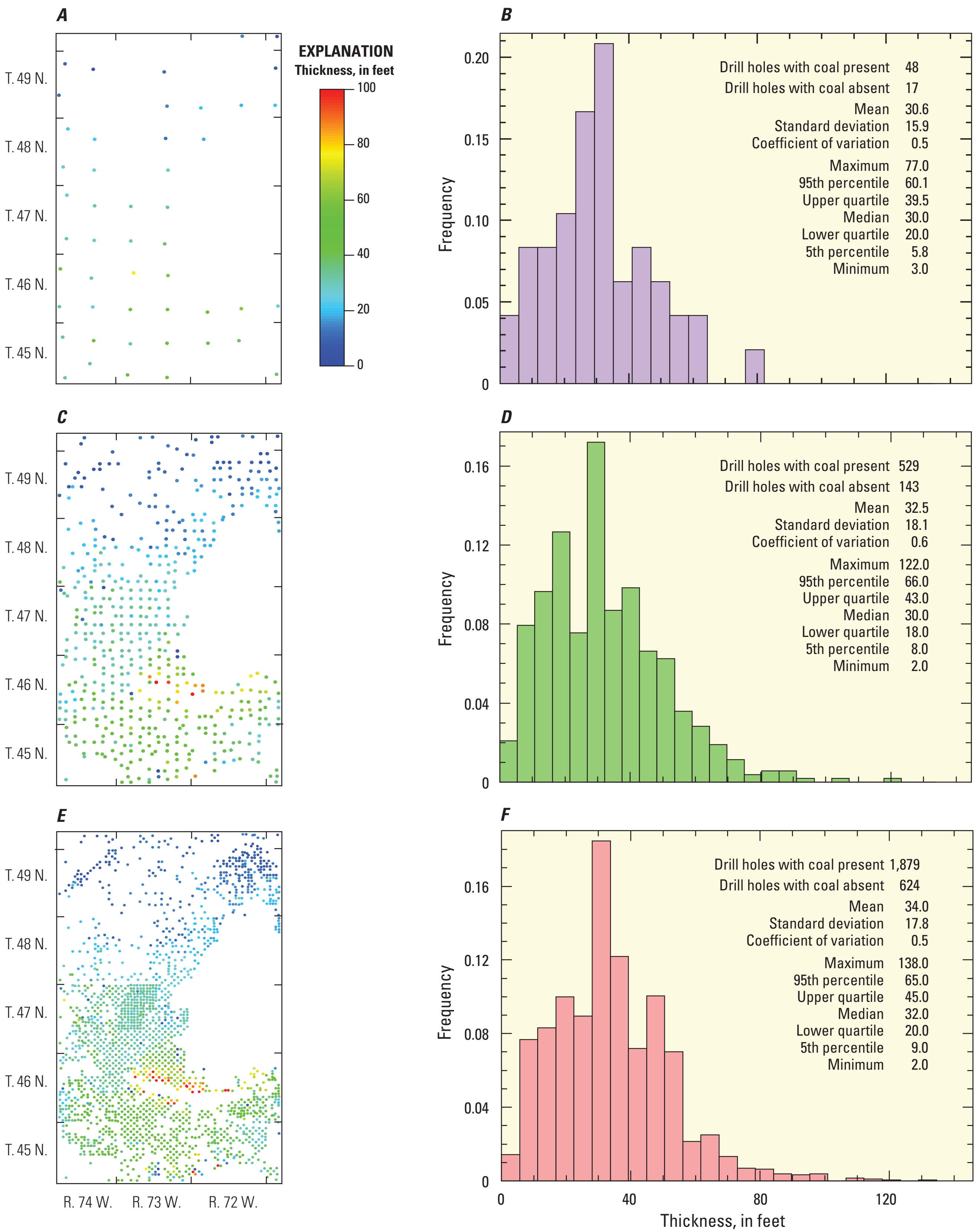

Figure 10. Thickness data for Canyon coal bed. *A*, Reconnaissance posting. *B*, Reconnaissance histogram. *C*, Infill posting. *D*, Infill histogram. *E*, Development posting. *F*, Development histogram.

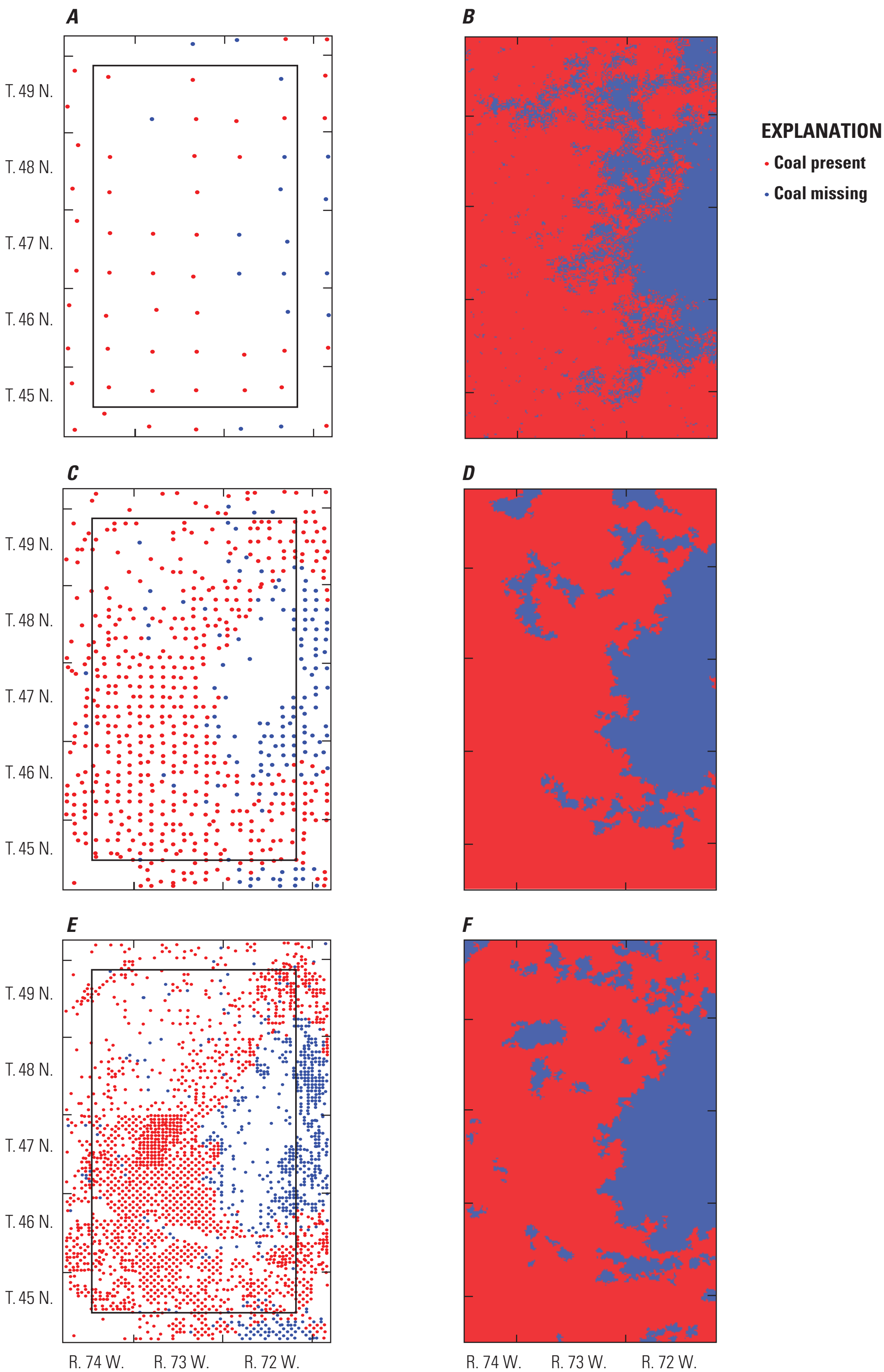

Figure 11. Presence and absence of the Canyon coal bed according to thickness indicators. The boundary for the realizations is the inner box. *A*, Reconnaissance posting. *B*, Reconnaissance first realization. *C*, Infill posting. *D*, Infill first realization. *E*, Development posting. *F*, Development first realization.

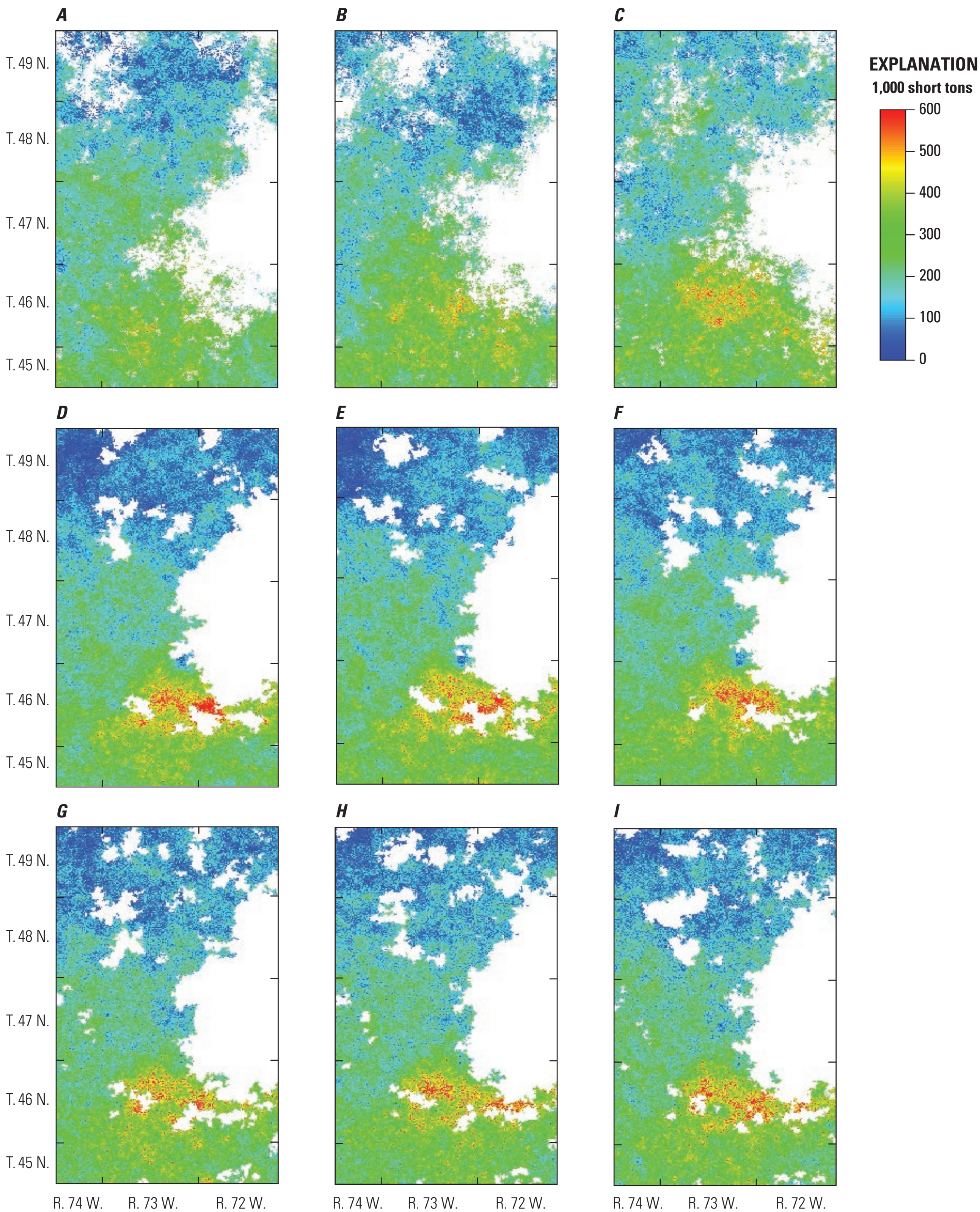

Figure 12. Tonnage realizations for the Canyon coal bed, selected percentiles. *A*, Reconnaissance 5th percentile. *B*, Reconnaissance median (50th percentile). *C*, Reconnaissance 95th percentile. *D*, Infill 5th percentile. *E*, Infill median. *F*, Infill 95th percentile. *G*, Development 5th percentile. *H*, Development median. *I*, Development 95th percentile.

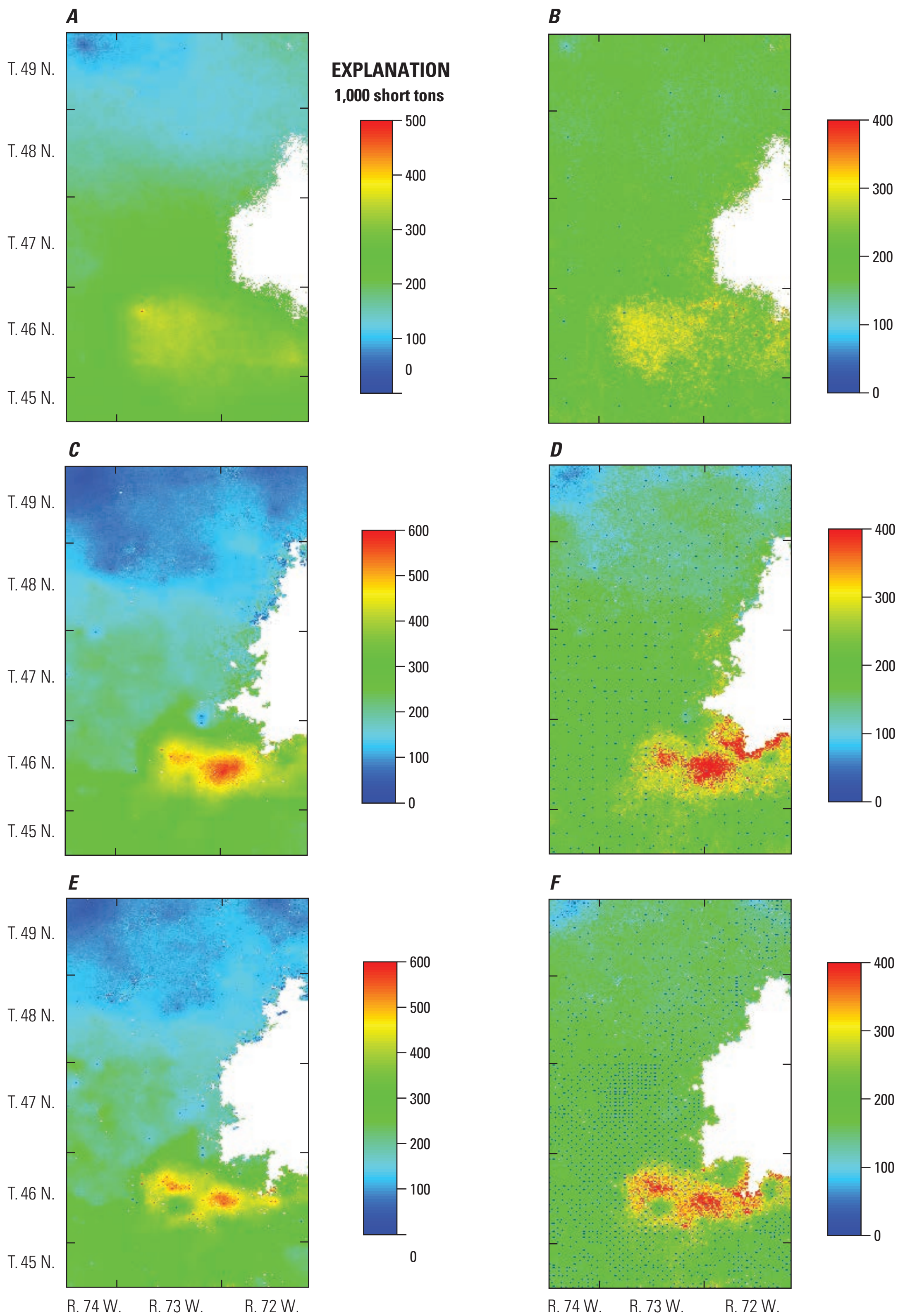

Figure 13. Maps of cell properties for Canyon coal bed, mean and 5–95 spread. *A*, Reconnaissance mean. *B*, Reconnaissance 5–95 spread. *C*, Infill mean. *D*, Infill 5–95 spread. *E*, Development mean. *F*, Development 5–95 spread.

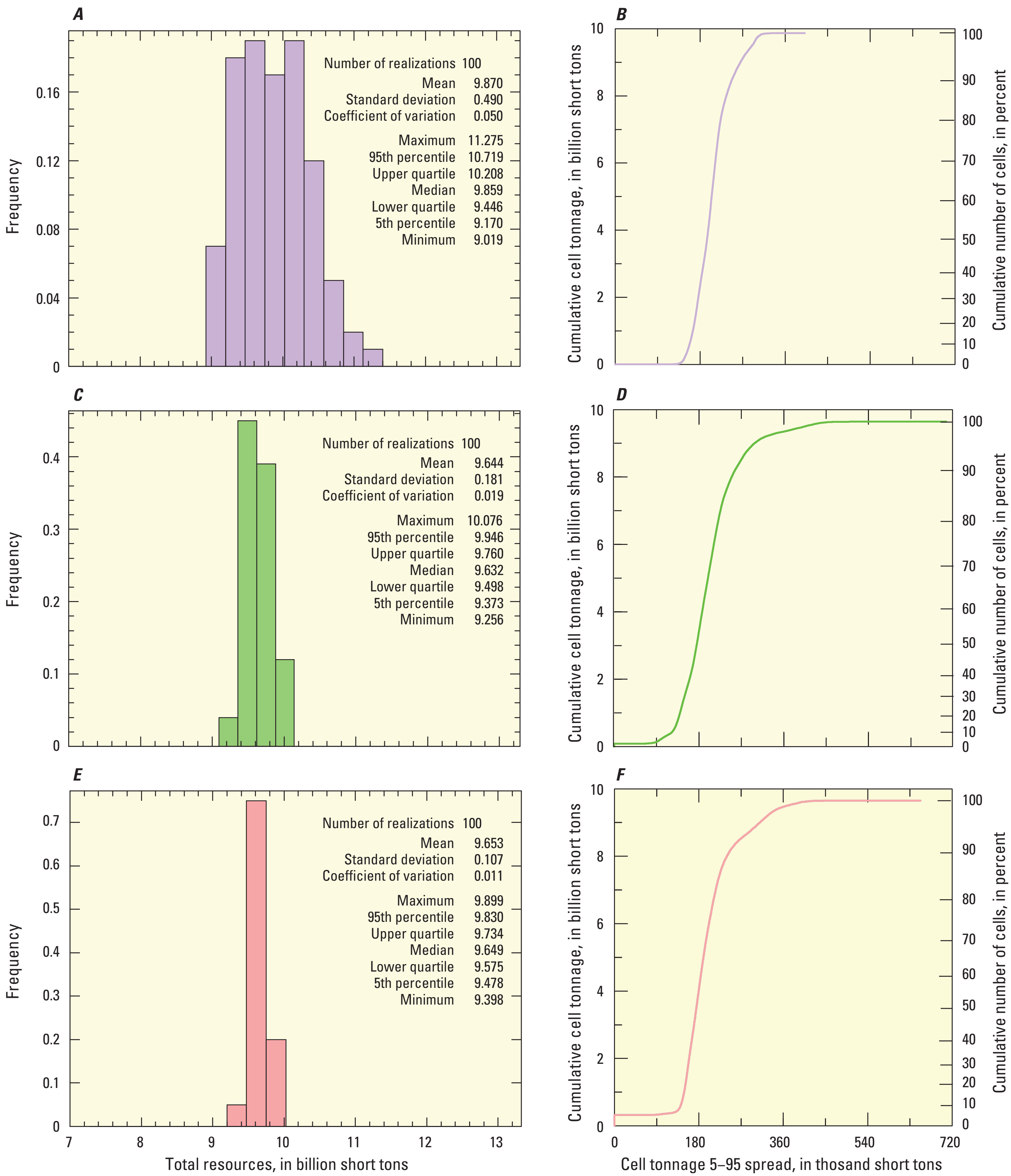

Figure 14. Summary of uncertainty in resources for the Canyon coal bed. *A*, Reconnaissance histogram of total tonnage. *B*, Reconnaissance cumulative tonnage as a function of cell tonnage uncertainty. *C*, Infill histogram of total tonnage. *D*, Infill cumulative tonnage as a function of cell tonnage uncertainty. *E*, Development histogram of total tonnage. *F*, Development cumulative tonnage as a function of cell tonnage uncertainty.

Anderson Coal Bed

The Anderson is the second coal bed from the bottom in the sequence being modeled and it is the thickest (figs. 8 and 9). Figure 15 shows the thickness values for all three drilling stages. Frequency content of all histograms is remarkably similar despite the orders of magnitude in the number of drill holes and the fact that the reconnaissance drilling again failed to detect the channel in T. 46 N., R. 73 W. Figure 16 shows the transformation from thickness values to presence-absence thickness indicators.

Figure 17 displays 3 of the total number of 100 realizations generated to model the resources differing from thickness solely by a conversion factor. The 5th percentile is representative of the most unfavorable scenarios, the median is a middle-of-the-road alternative, and the 95th percentile is typical of the most optimistic scenarios.

Figures 18 and 19 summarize results intended to quantify uncertainty in the modeling of the resource. This time, fluctuations in the 5–95 spread with drilling density are more typical of attempts to decrease the regular spacing. As seen in the right column of figures 18 and 19, there is a systematic reduction in 5−95 spread. The same figures also illustrate the significant amount of additional drilling required to reduce significantly the cell uncertainty. In this case, a roughly 45-fold increase in drill holes has resulted in a reduction of the 5−95 spread by a mere factor of approximately 2.

It is also interesting to note a change in the mixture of cells with different levels of uncertainty. For example, when the drilling density is low, it is possible to have cells with low uncertainty—a small 5−95 spread—only in those areas where the tonnage is consistently low. As the data increase—in this case there are no areas showing an increase in complexity—there is an increase in the number of cells with high tonnage that have as low, or lower, uncertainty that the reconnaissance cells with low uncertainty. For example, for reconnaissance, the 40 percent of cells with the lowest 5−95 spread contributed only 5.2 bst. For the infill drilling the same proportion of cells contributed 7.1 bst and for development the tonnage was even higher at 8.2 bst. The southwest corner remained the area with the lowest uncertainty. By using the 2,201 drill holes added during the development drilling to check the actual number of new values falling in the 5–95 spread, the proportion was 95.6 percent, implying that the confidence intervals in the modeling were slightly wide.

Smith Coal Bed

The Smith coal bed is the second youngest unit in the modeling and also the second thinnest (figs. 8 and 9). Figures 20 and 21 show the thickness locations and summary statistics. The main features of the coal bed are mostly missing in the northern end of the study area and there is clear thickening to the west.

Figure 22 displays a selection of scenarios for the Smith resource. Although the number of drill holes is the same as those for the other coal beds, the reconnaissance drilling has this time captured all important features of the spatial variability. The minor details impossible to capture with 77 drill holes are the isolated patches of the coal bed toward the north where it is mostly, but not completely, absent.

The most notable aspect of the maps in figure 23 is the close association between mean cell tonnage and the 5−95 spread. For most of the area, there is a direct dependency, but the spread is only intermediate in magnitude for the highest values in the southwest edge of the study area. In the right column of figure 24, the association translates into low contribution to total resources by the more accurate cells. For example, the 40 percent of the less uncertain cells contributes only about 1 bst, which is only 15 percent of total tonnage. Validation of 5−95 spread confidence by using the 1,935 new development values reveals that 94.2 percent of the values are within the intervals instead of the theoretical 90 percent coverage.

Roland Coal Bed

The Roland coal bed is close enough to the surface to suspect that it may have undergone oxidation, if oxidation depth is assumed to be 35 feet. Consequently, the bed was modeled by applying Procedure J in the toolkit section (at back of report). As is evident in figure 25, the Roland is also the thinnest of all coal beds in this study and, from figure 26, its presence throughout the modeled area is comparable to that of the Anderson.

We assumed that there will be oxidation wherever the roof of the coal bed is shallower than 35 feet. Modeling of such a conditional situation requires mapping the surface elevation of the coal bed roof, which, similar to thickness, is only known at the drill-hole intersections. On the contrary, the land-surface elevation is known without error (fig. 27). We used the same approach to model uncertainty in the oxidation by generating 100 realizations of the roof (fig. 28). The outcome is that the oxidation is minimal, but not null. Application of Procedure J was useful, however, primarily to illustrate the flexibility in the modeling to adjust to the realities of the geology.

Figure 29 displays 3 of the 100 realizations per drilling stage containing all the information necessary to prepare the modeling of uncertainty summarized in the following two figures.

As with the other three, deeper coal beds, figure 30 shows that an increase in the number of drill holes results in a reduction in total resources uncertainty, with the range of possible values for the drilling with fewer holes completely including the narrower range associated with the denser drilling, denoting a convergence toward the true tonnage. Uncertainty also follows a tendency observed for the other coal beds. As the drilling density increases, the cumulative curve shifts to the left, denoting a general reduction in the width of the 5−95 spread. For example, for a cumulative value of 2.0 bst, the spread is 88 kst for reconnaissance, 70 kst for infill, and 61 kst for development (fig. 31). Note that the reduction of 19 percent is disproportionately small relative to the 45-fold increase in the number of drill holes. The infill case shows another interesting situation observed previously: the higher drilling density sometimes results in significant fluctuations in tonnage within short distances, thus increasing uncertainty despite the increase in data, which is the case of the anomalously large 5−95 spread value at the high end of the infill distribution. Validation of the 5−95 spread performance using the additional 2,173 development drill holes resulted in an actual coverage of 94.4 percent instead of the 90 percent nominal coverage. Although the discrepancy is acceptable, slightly narrower intervals would have yielded a perfect match with the theoretical value.

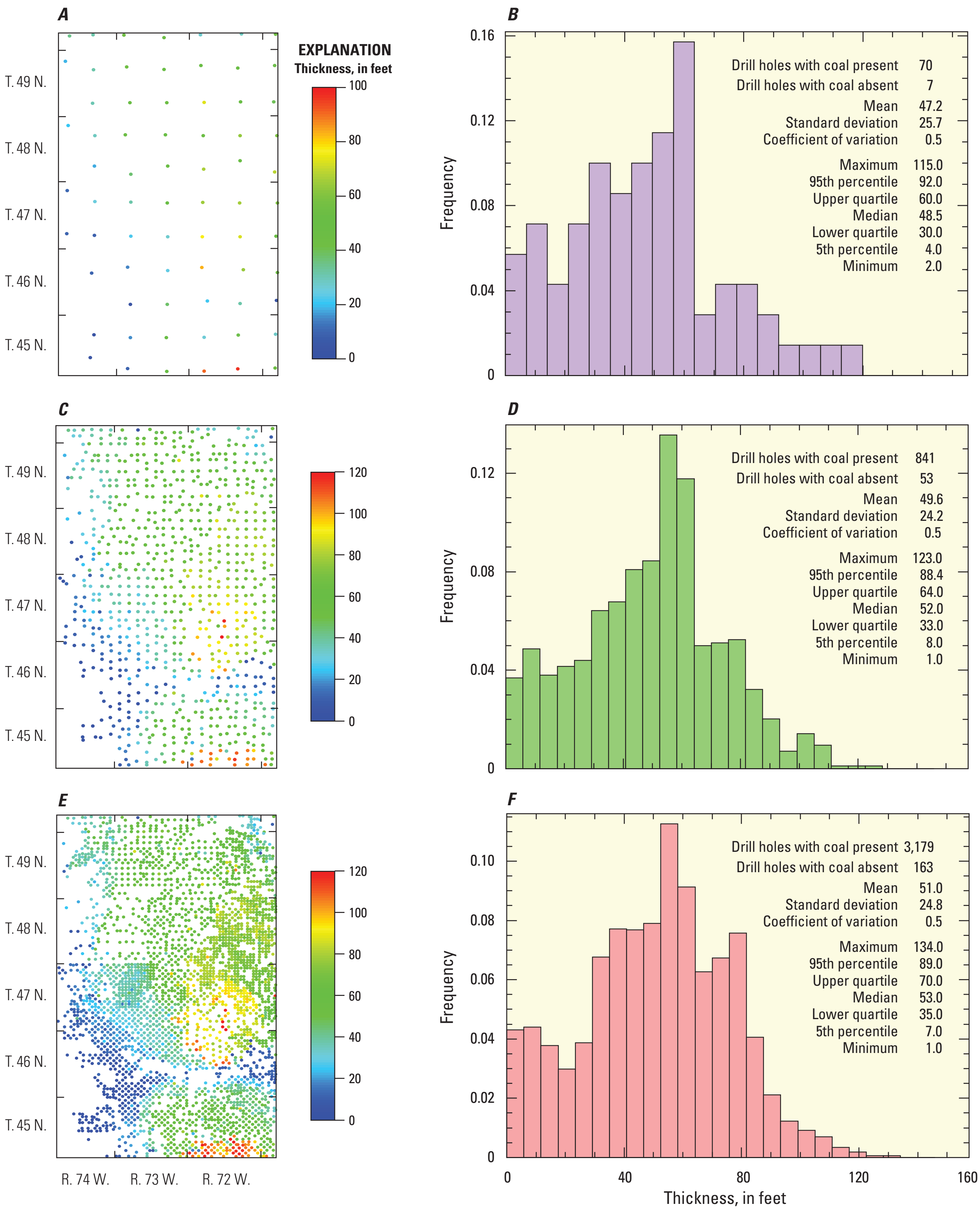

Figure 15. Thickness data for the Anderson coal bed. *A*, Reconnaissance posting. *B*, Reconnaissance histogram. *C*, Infill posting. *D*, Infill histogram. *E*, Development posting. *F*, Development histogram.

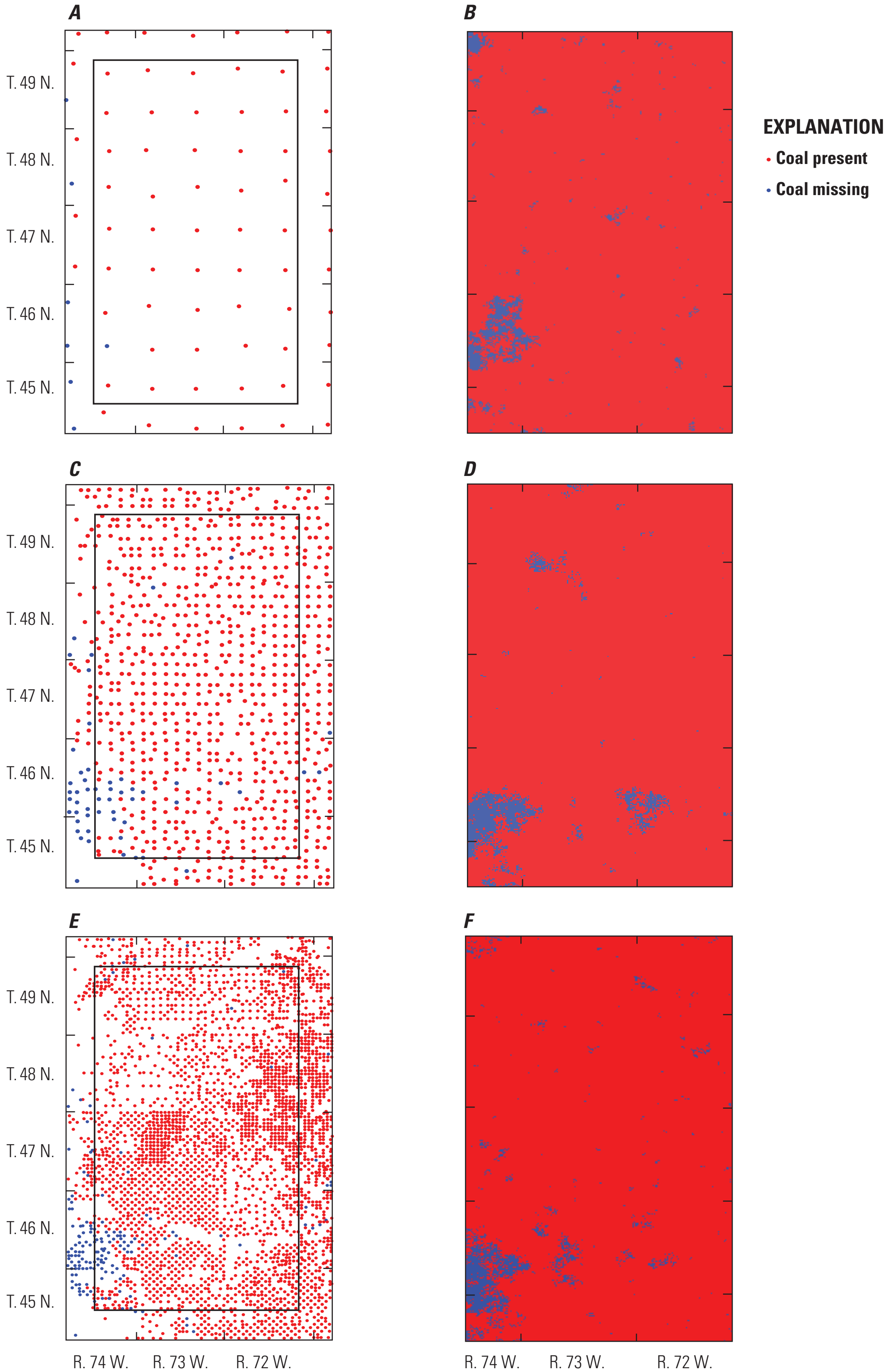

Figure 16. Presence and absence of the Anderson coal bed according to thickness indicators. The boundary for the realizations is the inner box. *A*, Reconnaissance posting. *B*, Reconnaissance first realization. *C*, Infill posting. *D*, Infill first realization. *E*, Development posting. *F*, Development first realization.

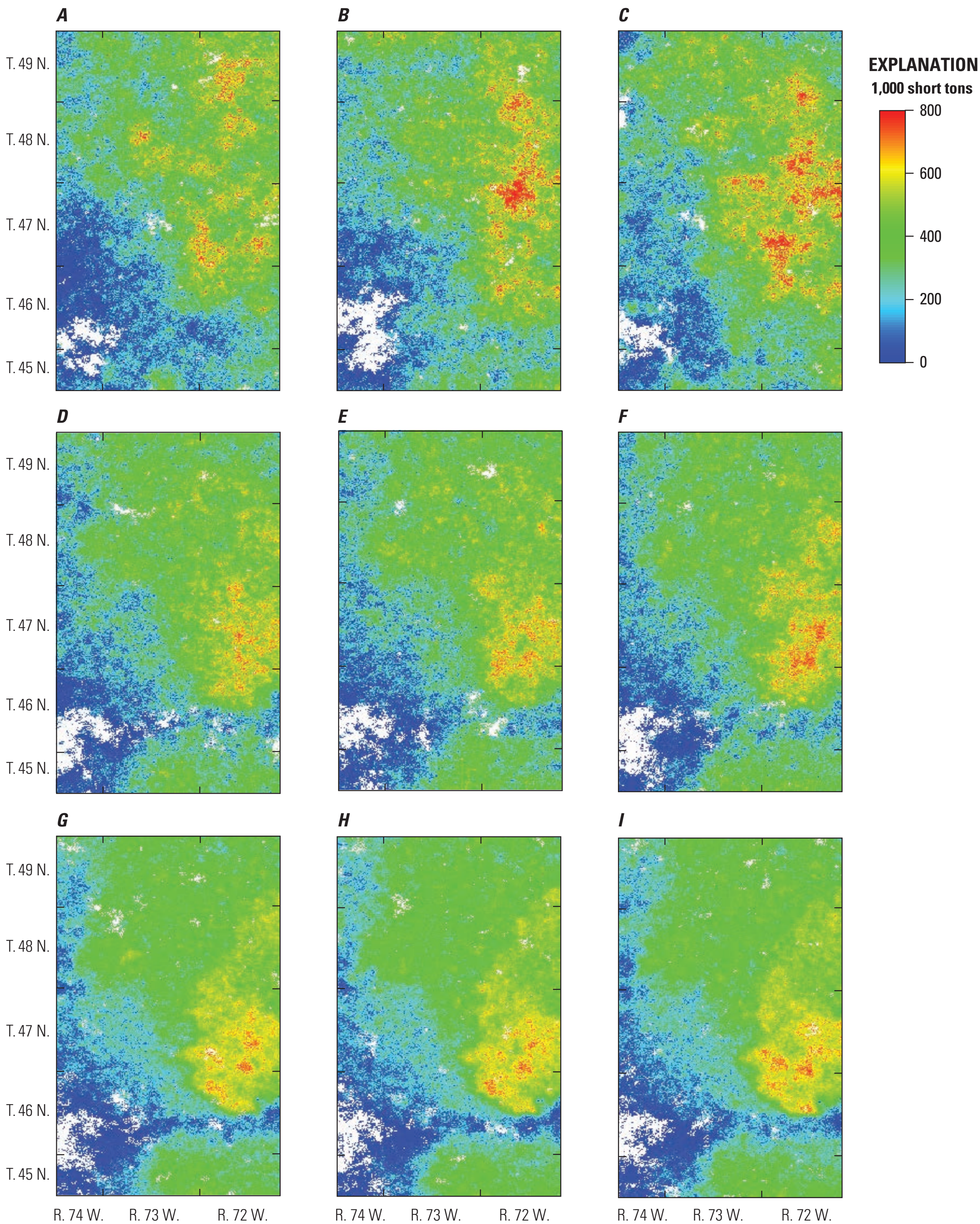

Figure 17. Tonnage realizations for the Anderson coal bed, selected percentiles. *A*, Reconnaissance 5th percentile. *B*, Reconnaissance median (50th percentile). *C*, Reconnaissance 95th percentile. *D*, Infill 5th percentile. *E*, Infill median. *F*, Infill 95th percentile. *G*, Development 5th percentile. *H*, Development median. *I*, Development 95th percentile.

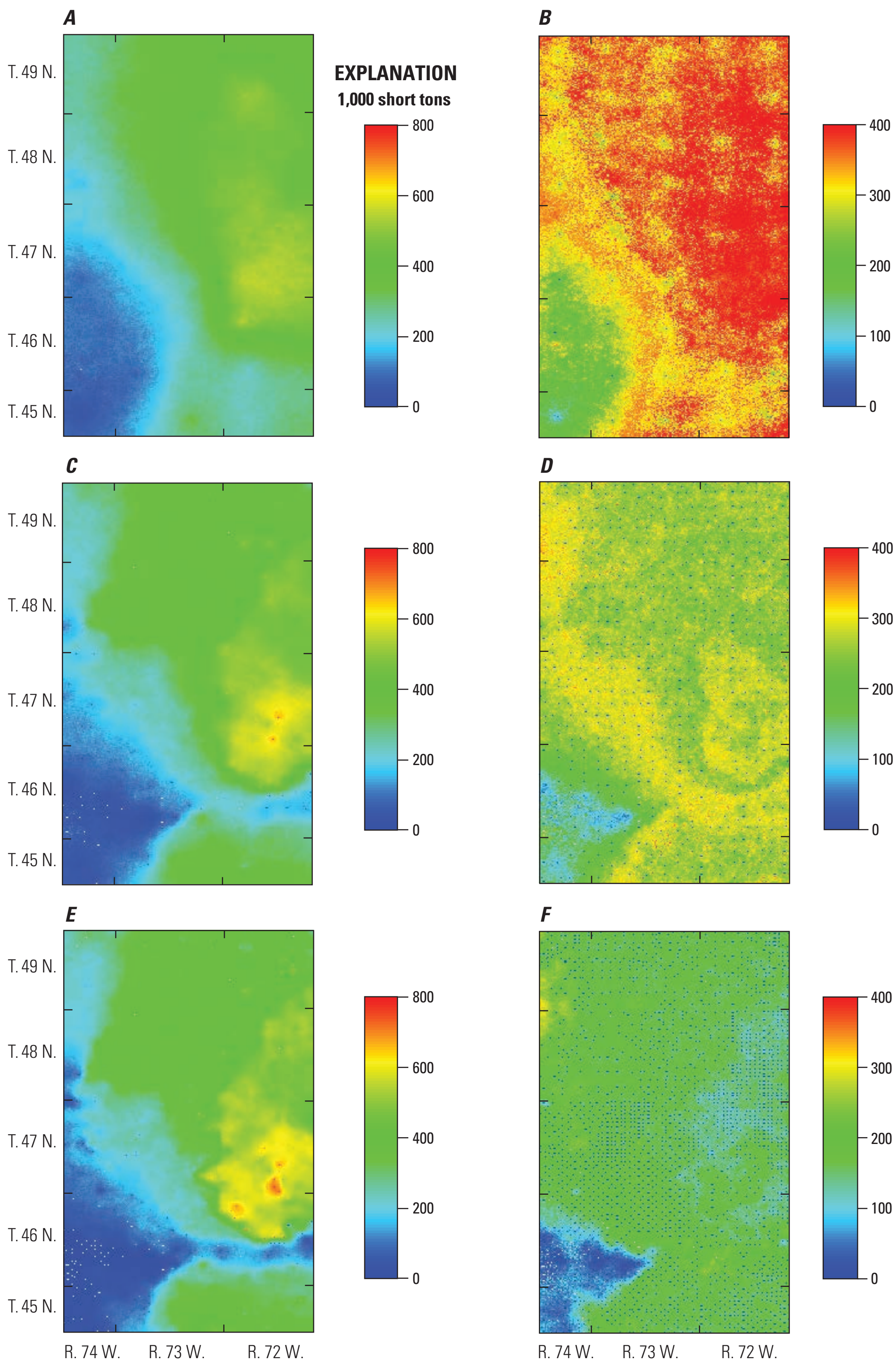

Figure 18. Maps of cell properties for Anderson coal bed, mean and 5–95 spread. *A*, Reconnaissance mean. *B*, Reconnaissance 5–95 spread. *C*, Infill mean. *D*, Infill 5–95 spread. *E*, Development mean. *F*, Development 5–95 spread.

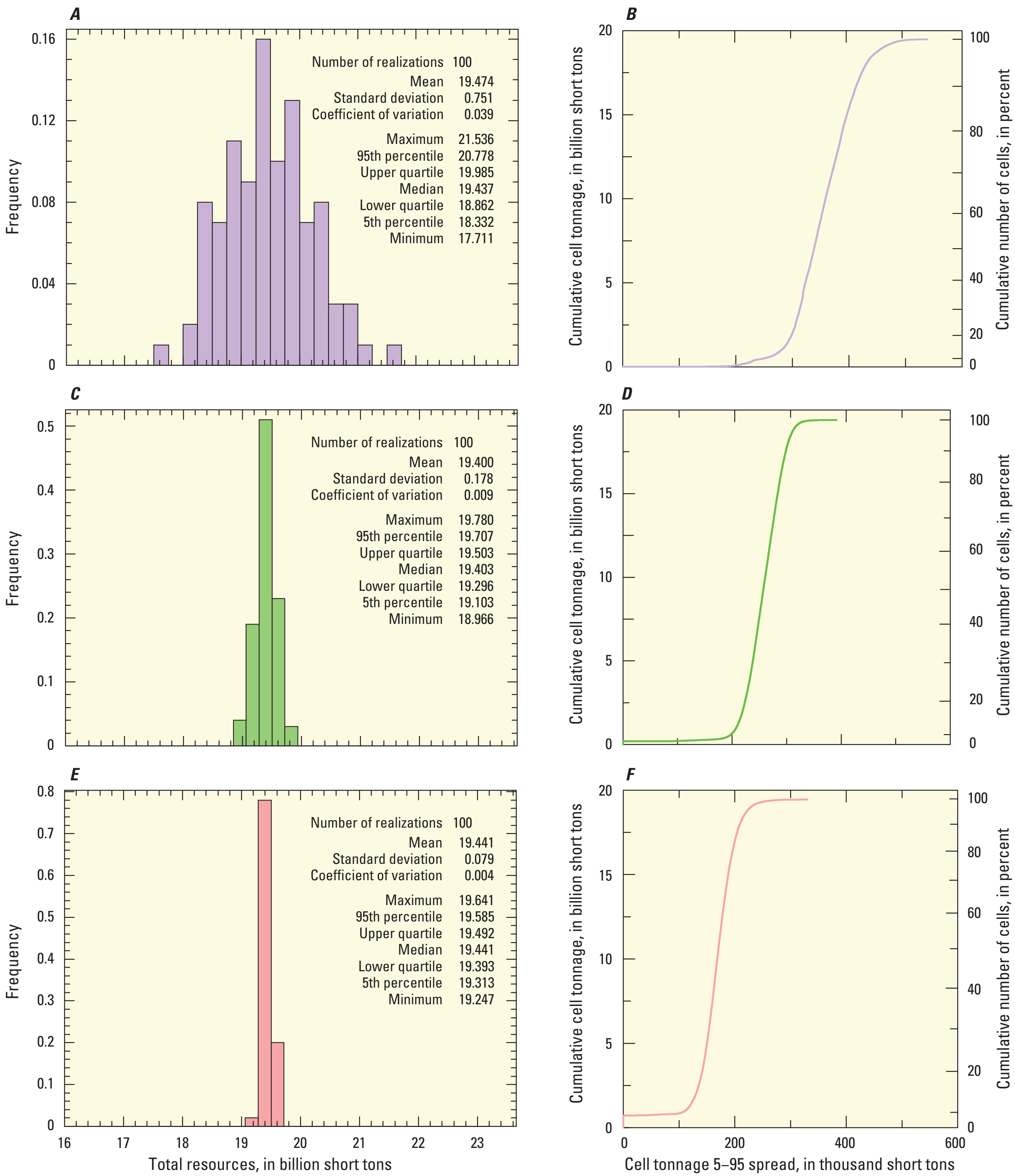

Figure 19. Summary of uncertainty in resources for the Anderson coal bed. *A*, Reconnaissance histogram of total tonnage. *B*, Reconnaissance cumulative tonnage as a function of cell tonnage uncertainty. *C*, Infill histogram of total tonnage. *D*, Infill cumulative tonnage as a function of cell tonnage uncertainty. *E*, Development histogram of total tonnage. *F*, Development cumulative tonnage as a function of cell tonnage uncertainty.

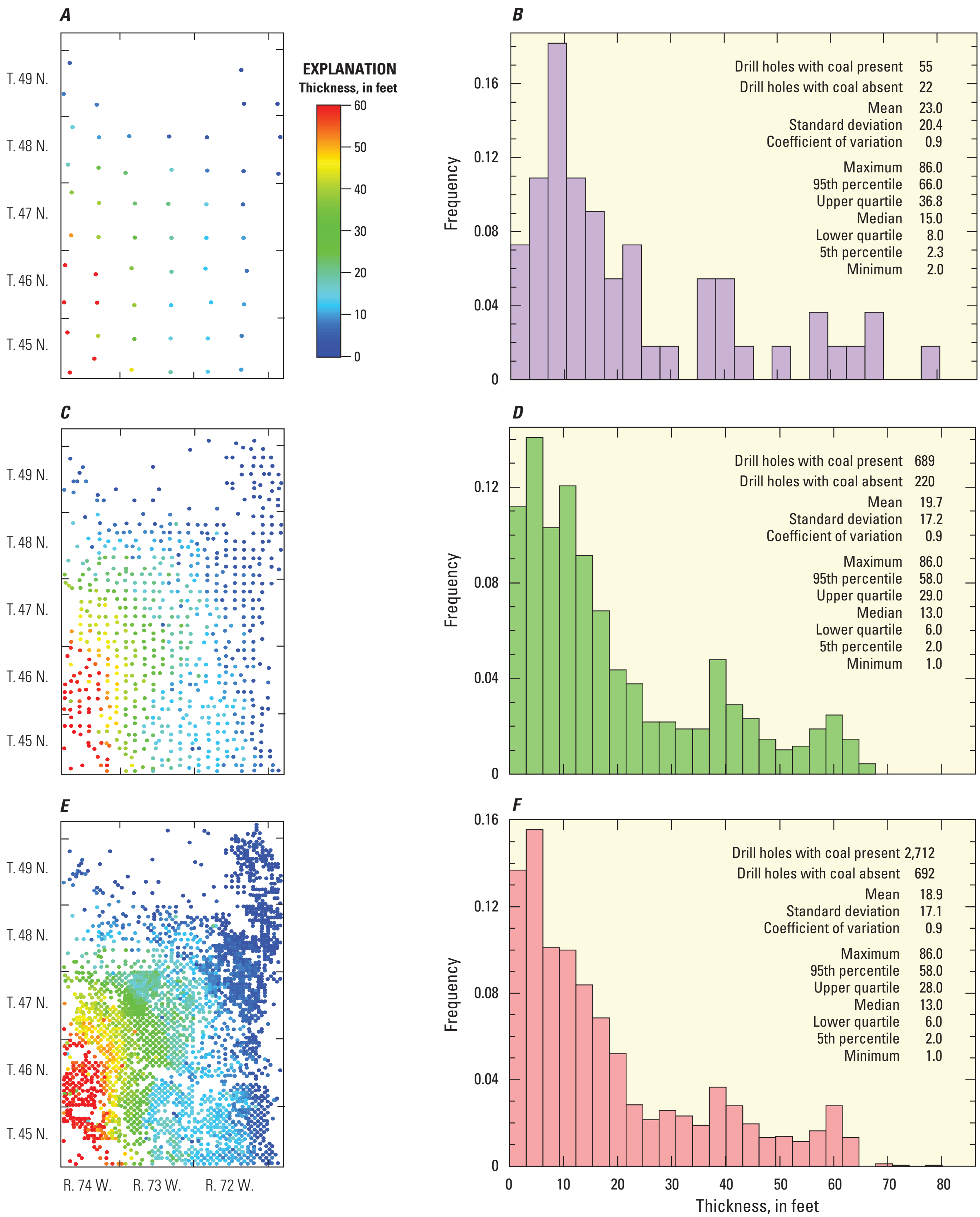

Figure 20. Thickness data for the Smith coal bed. *A*, Reconnaissance posting. *B*, Reconnaissance histogram. *C*, Infill posting. *D*, Infill histogram. *E*, Development posting. *F*, Development histogram.

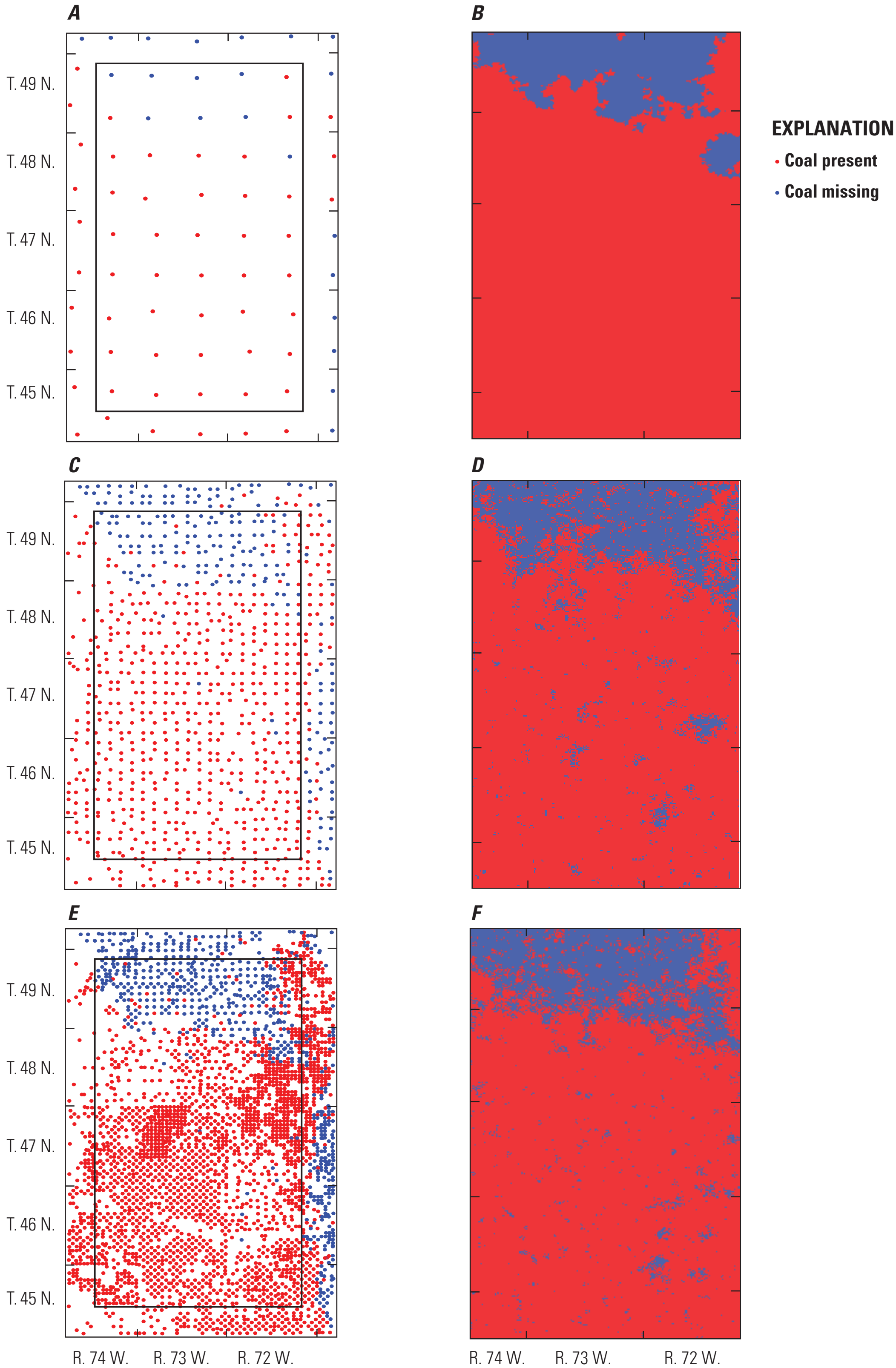

Figure 21. Presence and absence of the Smith coal bed according to thickness indicators. The boundary for the realizations is the inner box. *A*, Reconnaissance posting. *B*, Reconnaissance first realization. *C*, Infill posting. *D*, Infill first realization. *E*, Development posting. *F*, Development first realization.

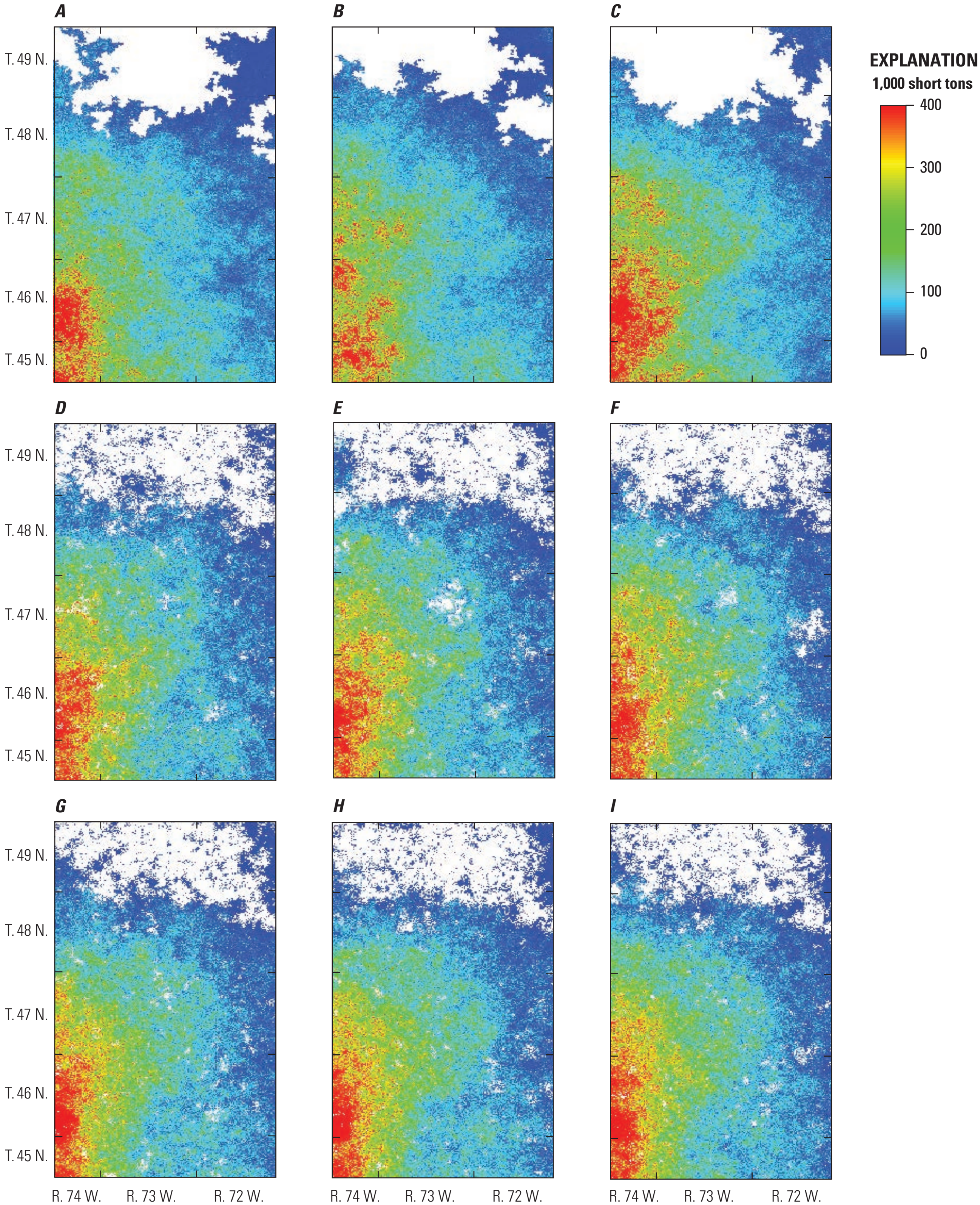

Figure 22. Tonnage realizations for the Smith coal bed, selected percentiles. *A*, Reconnaissance 5th percentile. *B*, Reconnaissance median (50th percentile). *C*, Reconnaissance 95th percentile. *D*, Infill 5th percentile. *E*, Infill median. *F*, Infill 95th percentile. *G*, Development 5th percentile. *H*, Development median. *I*, Development 95th percentile.

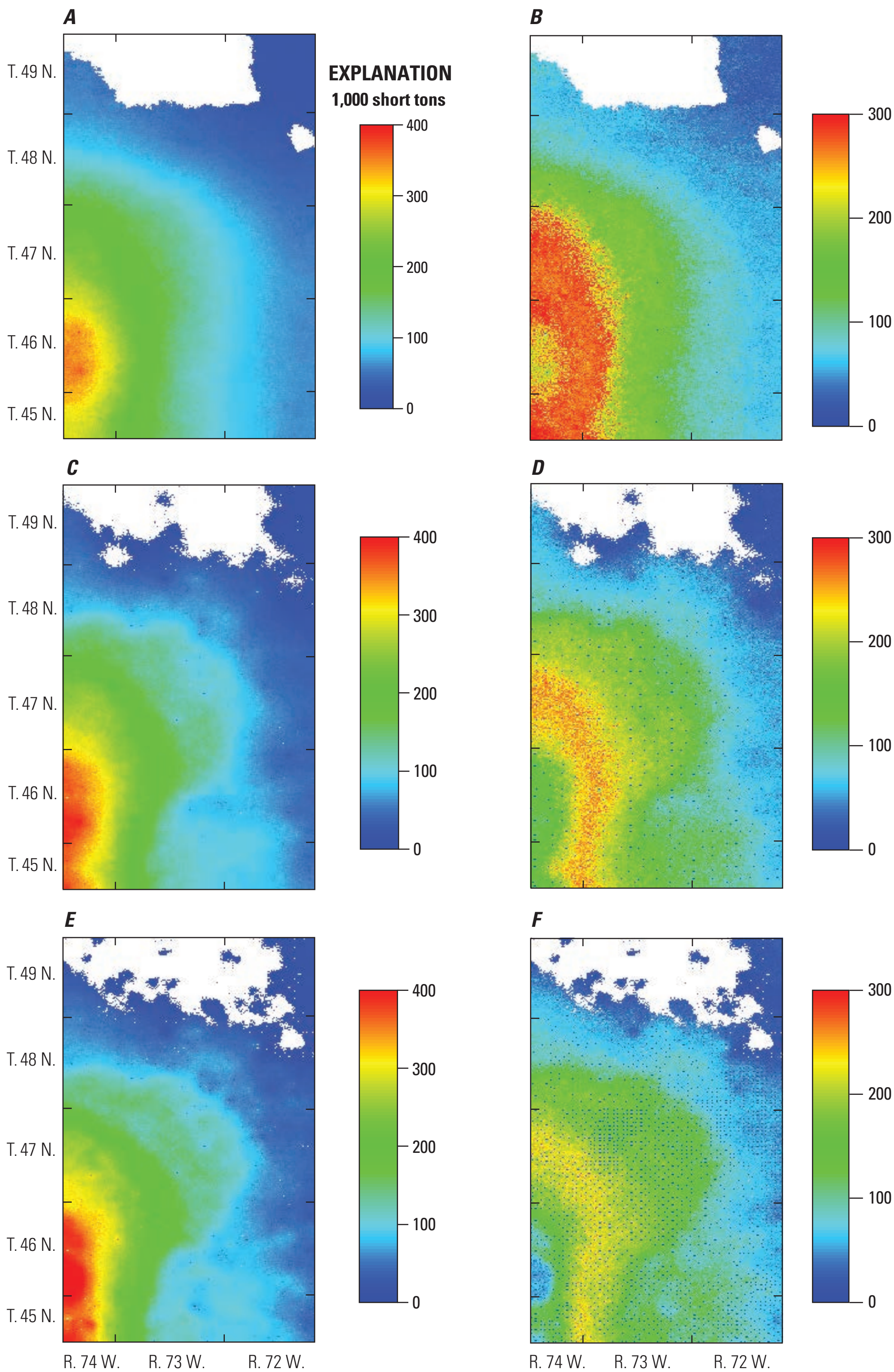

Figure 23. Maps of cell properties for Smith coal bed, mean and 5–95 spread. *A*, Reconnaissance mean. *B*, Reconnaissance 5–95 spread. *C*, Infill mean. *D*, Infill 5–95 spread. *E*, Development mean. *F*, Development 5–95 spread.

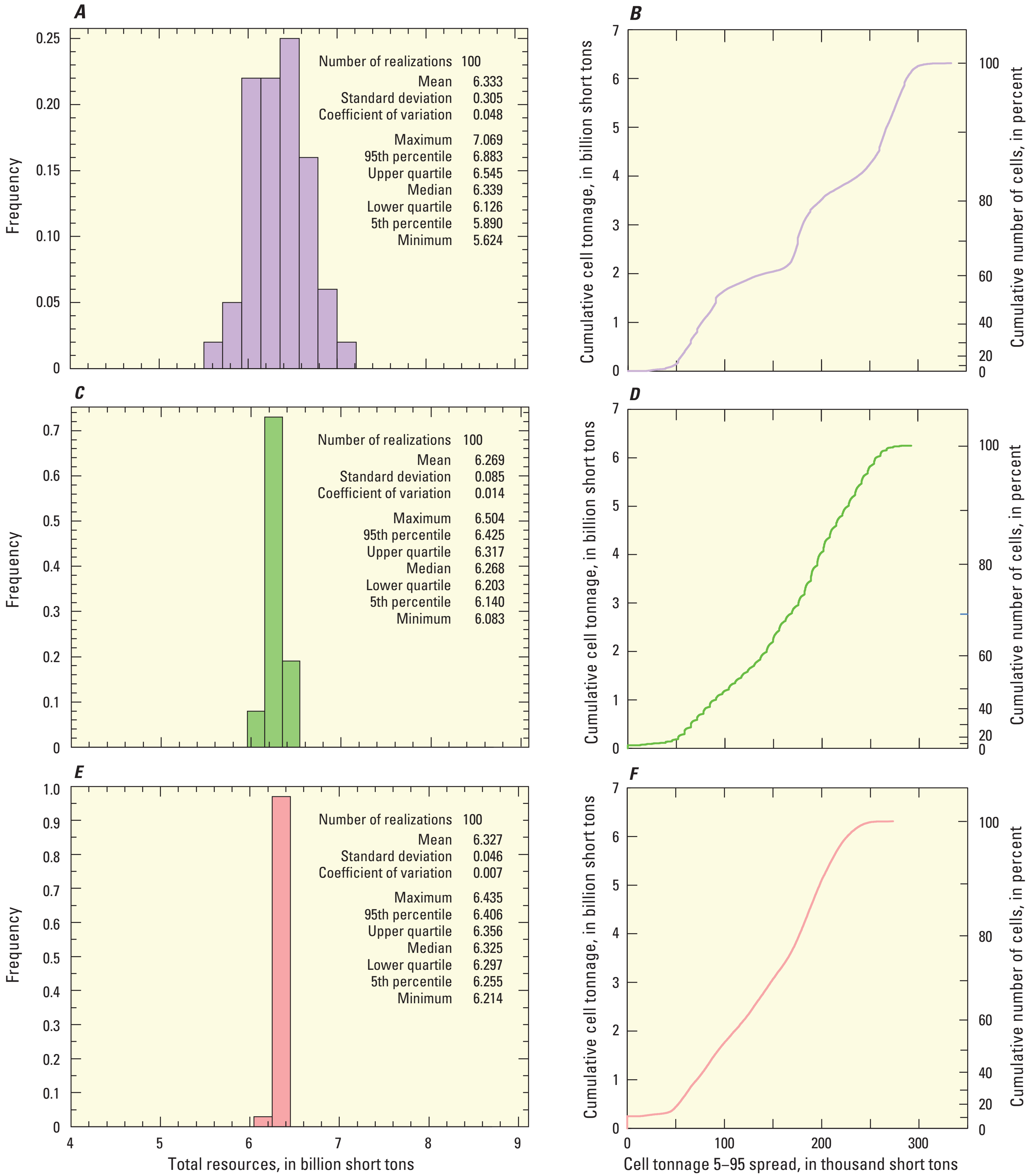

Figure 24. Summary of uncertainty in resources for the Smith coal bed. *A*, Reconnaissance histogram of total tonnage. *B*, Reconnaissance cumulative tonnage as a function of cell tonnage uncertainty. *C*, Infill histogram of total tonnage. *D*, Infill cumulative tonnage as a function of cell tonnage uncertainty. *E*, Development histogram of total tonnage. *F*, Development cumulative tonnage as a function of cell tonnage uncertainty.

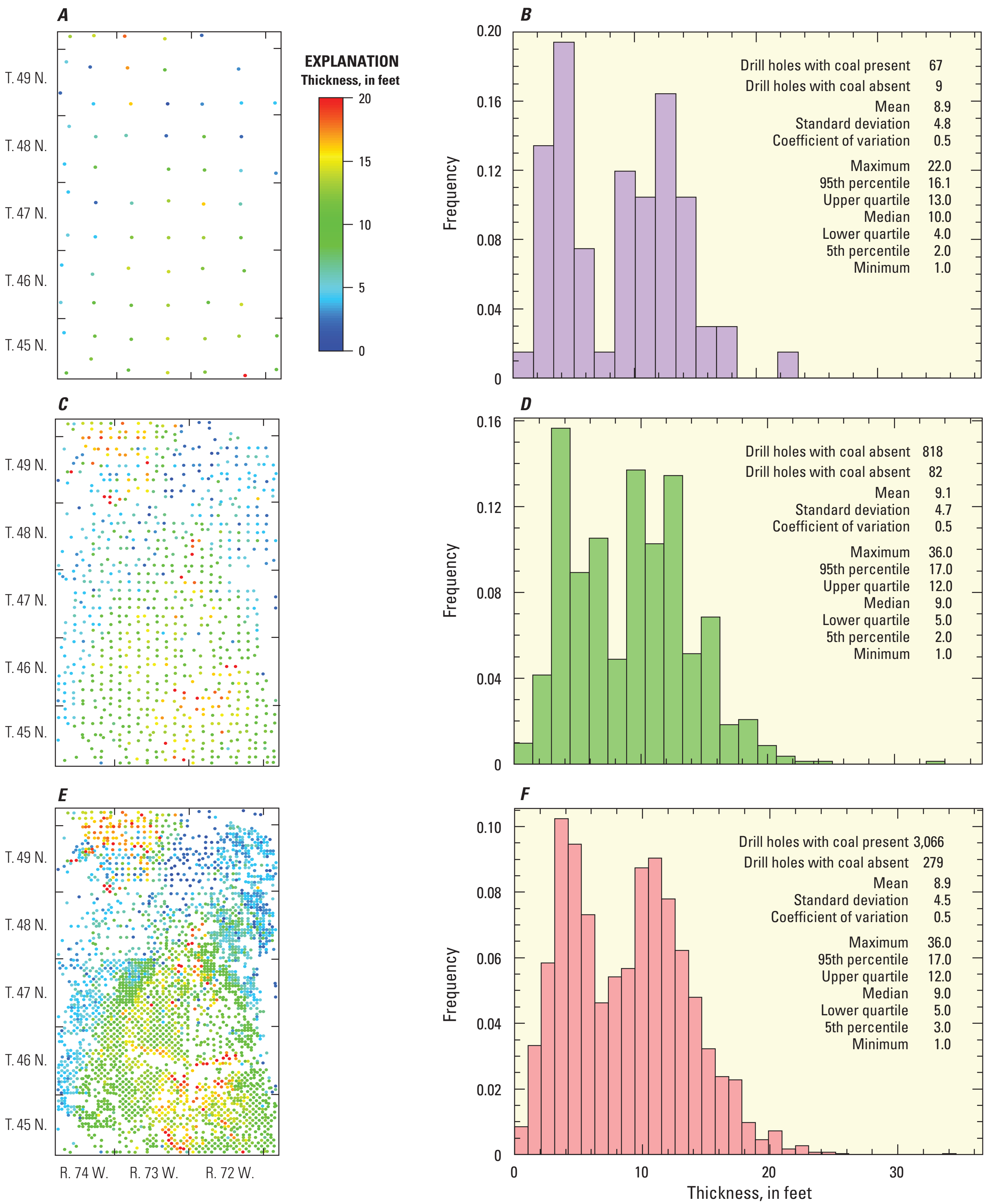

Figure 25. Thickness data for the Roland coal bed. *A*, Reconnaissance posting. *B*, Reconnaissance histogram. *C*, Infill posting. *D*, Infill histogram. *E*, Development posting. *F*, Development histogram.

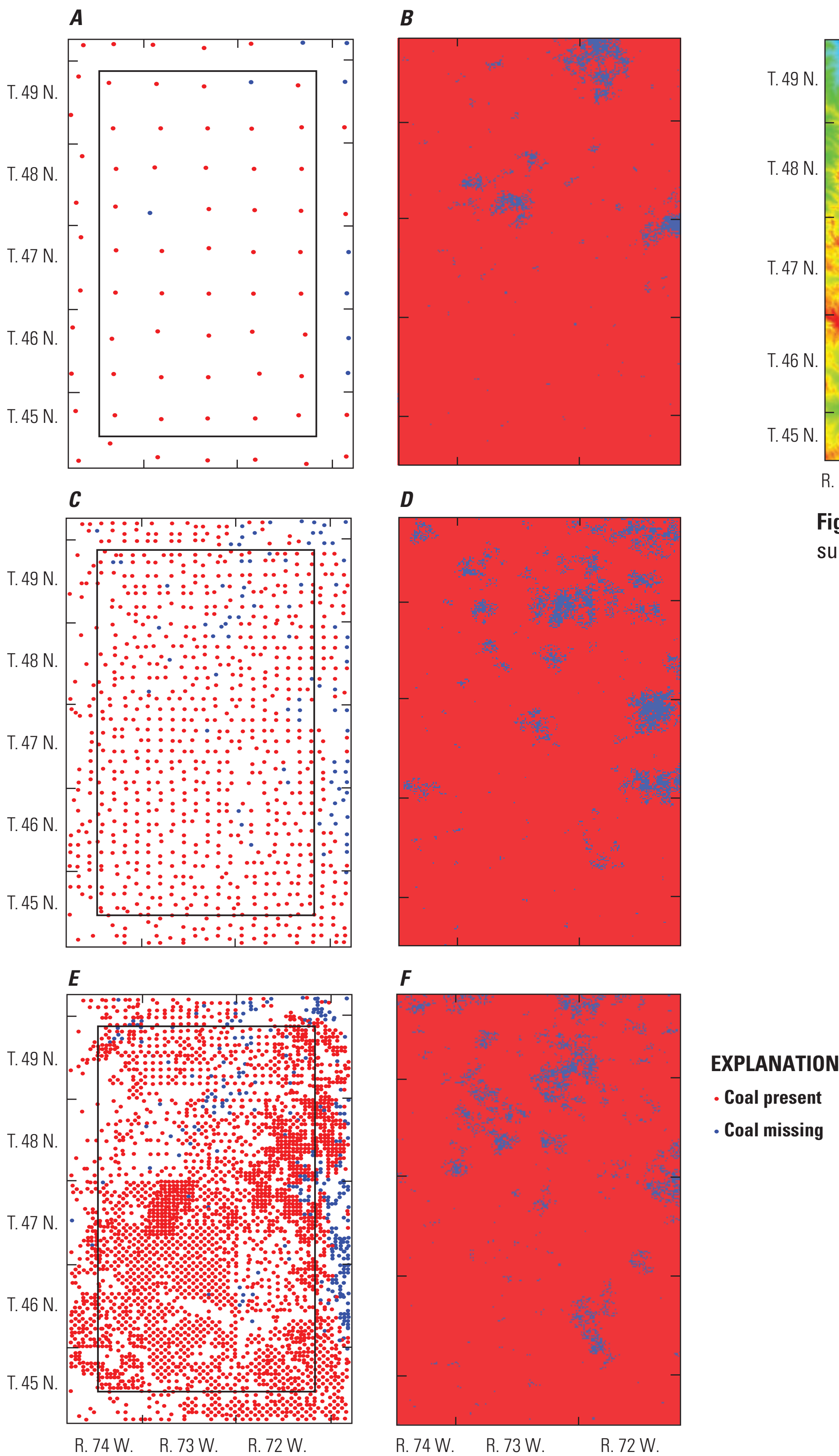

Figure 26. Presence and absence of the Roland coal bed according to thickness indicators. The boundary for the realizations is the inner box. *A*, Reconnaissance posting. *B*, Reconnaissance first realization. *C*, Infill posting. *D*, Infill first realization. *E*, Development posting. *F*, Development first realization.

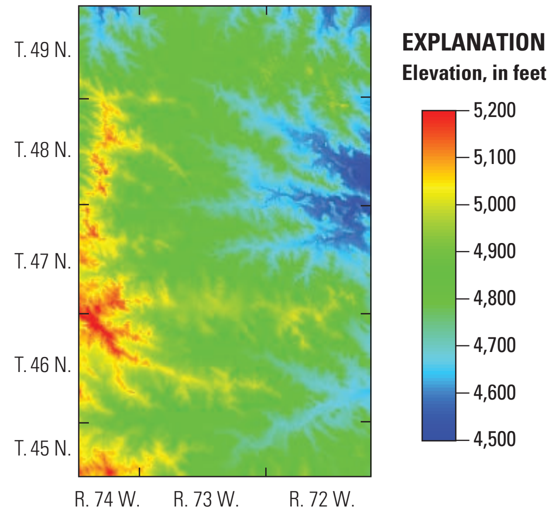

Figure 27. Digital model of land-surface elevation in the study area.

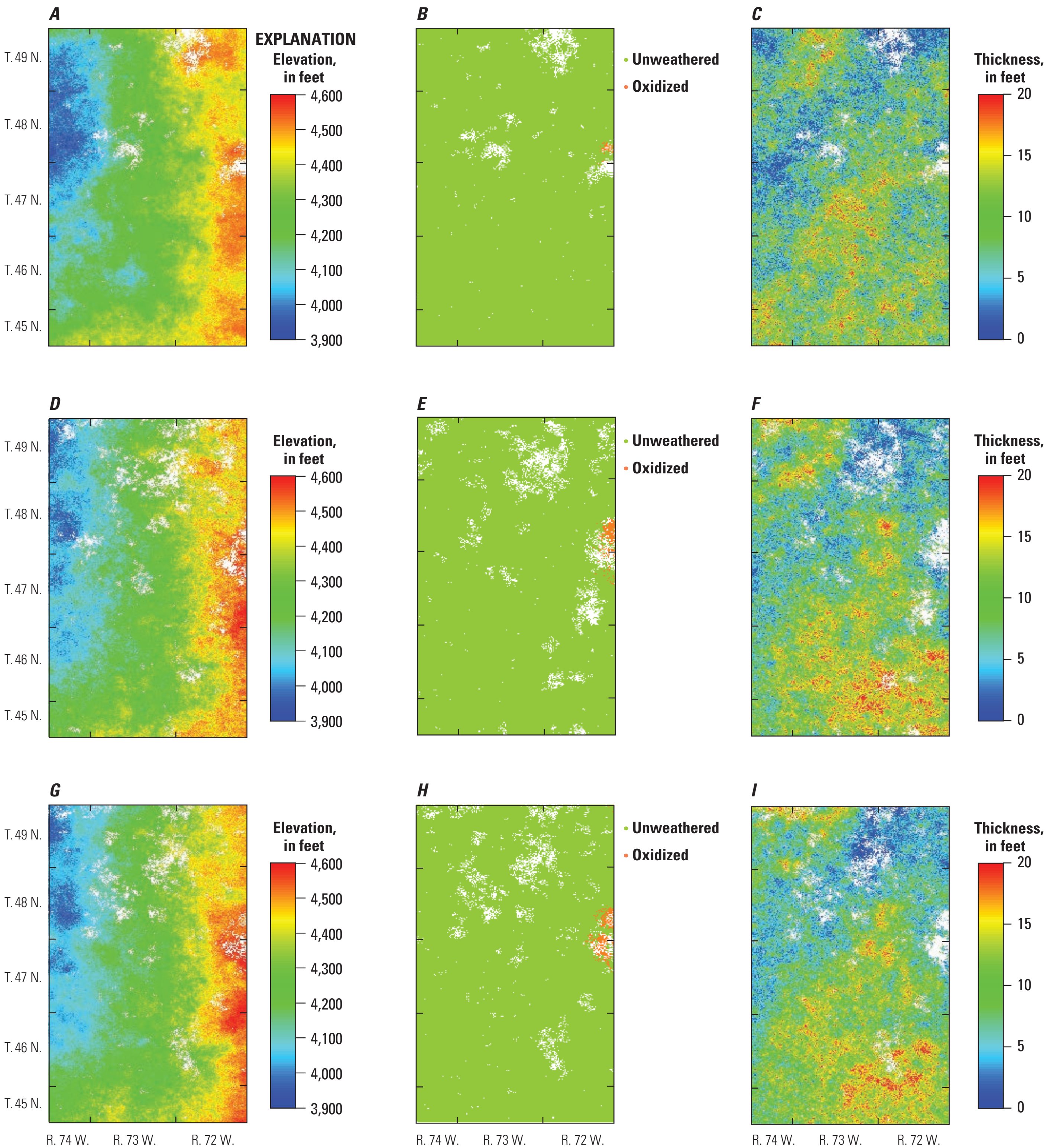

Figure 28. Modeling of oxidation of the Roland coal bed. *A*, Reconnaissance first realization of roof elevation. *B*, Reconnaissance first realization of oxidation indicator. *C*, Reconnaissance first realization of thickness of unoxidized coal. *D*, infill first realization of roof elevation. *E*, Infill first realization of oxidation indicator. *F*, Infill first realization of thickness of unoxidized coal. *G*, Development first realization of roof elevation. *H*, Infill first realization of oxidation indicator. *I*, Development first realization of thickness of unoxidized coal.

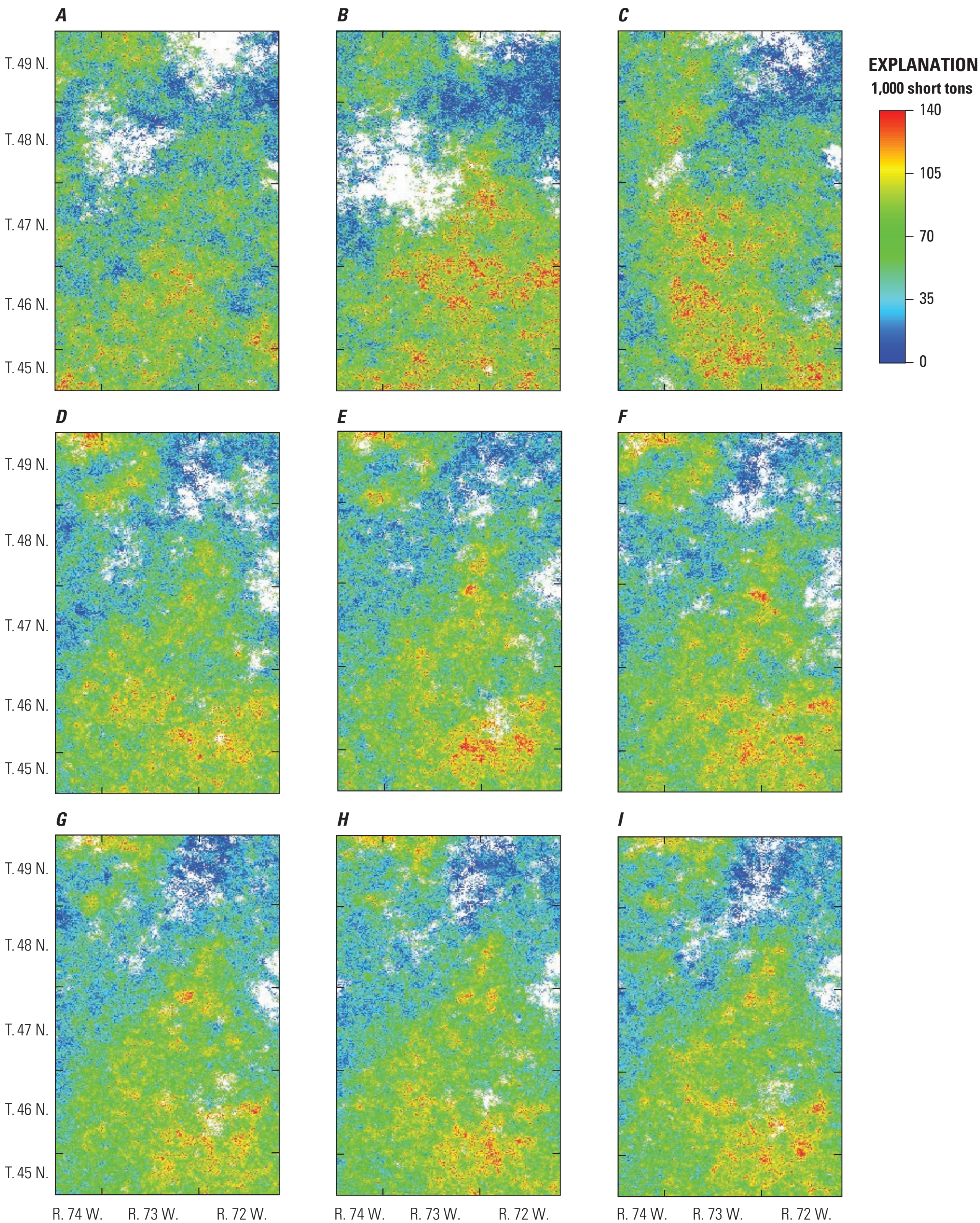

Figure 29. Tonnage realizations for the Roland coal bed, selected percentiles. *A*, Reconnaissance 5th percentile. *B*, Reconnaissance median (50th percentile). *C*, Reconnaissance 95th percentile. *D*, Infill 5th percentile. *E*, Infill median. *F*, Infill 95th percentile. *G*, Development 5th percentile. *H*, Development median. *I*, Development 95th percentile.

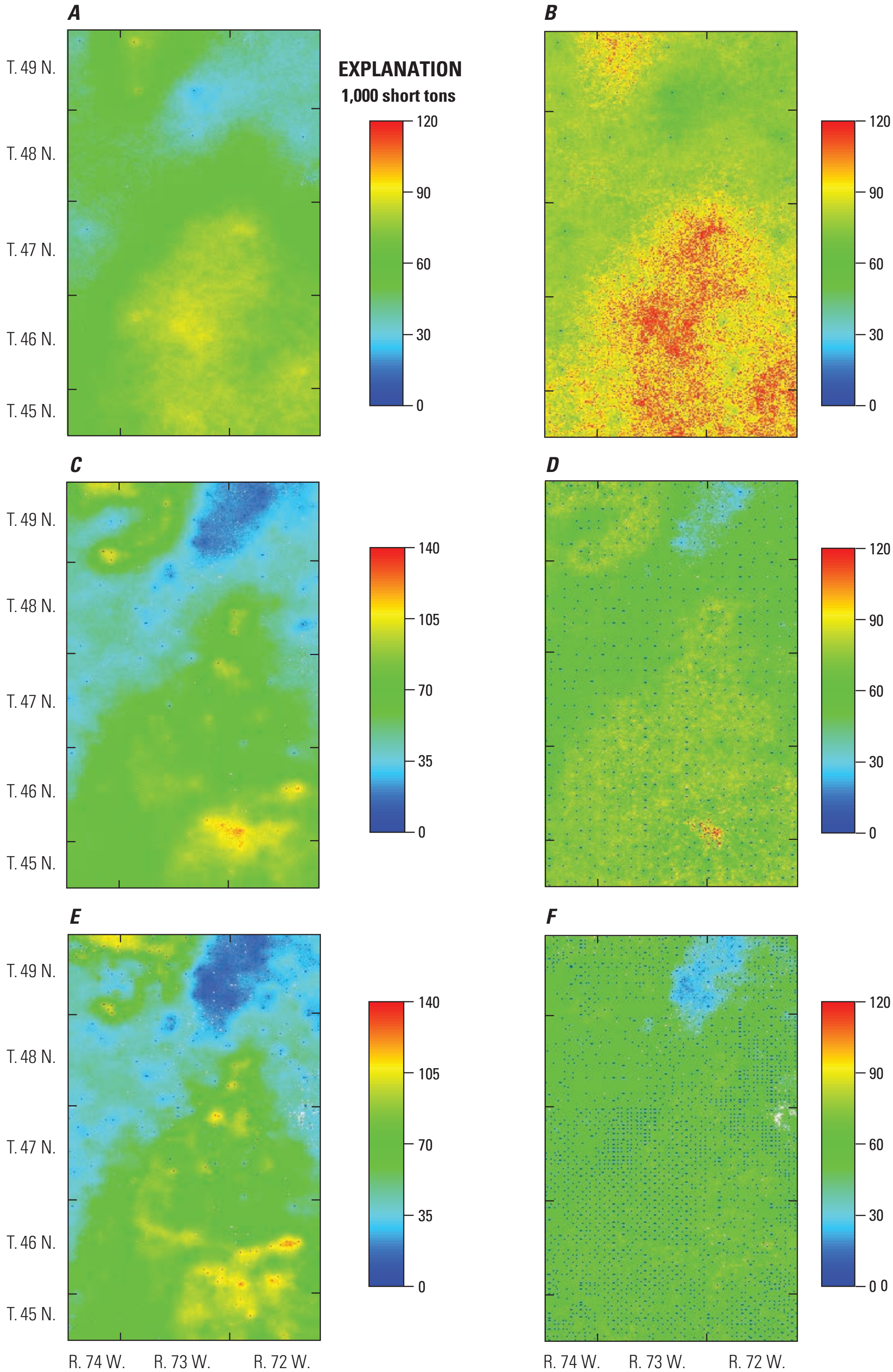

Figure 30. Maps of cell properties for Roland coal bed, mean and 5–95 spread. *A*, Reconnaissance mean. *B*, Reconnaissance 5–95 spread. *C*, Infill mean. *D*, Infill 5–95 spread. *E*, Development mean. *F*, Development 5–95 spread.

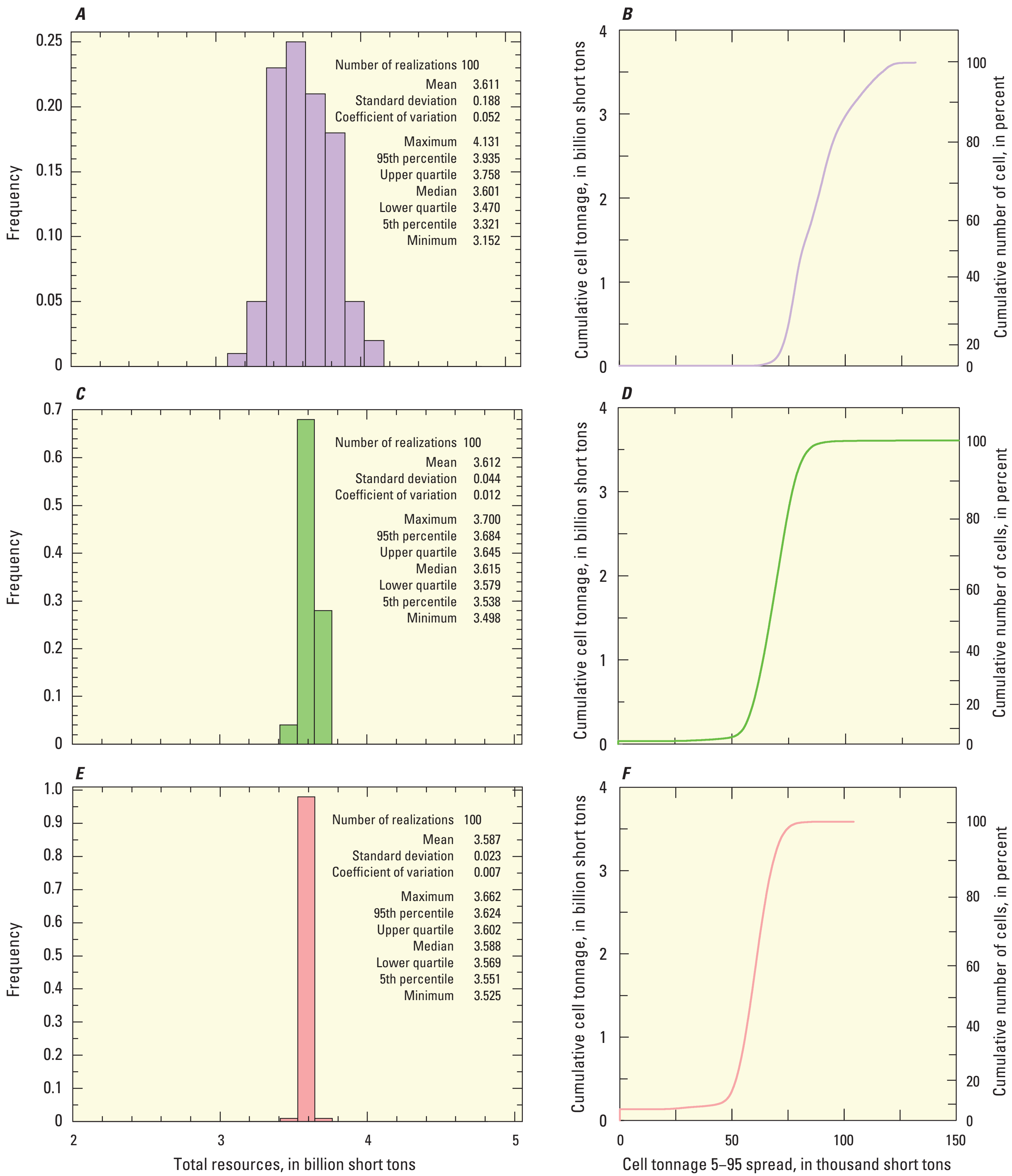

Figure 31. Summary of uncertainty in resources for the Roland coal bed. *A*, Reconnaissance histogram of total tonnage. *B*, Reconnaissance cumulative tonnage as a function of cell tonnage uncertainty. *C*, Infill histogram of total tonnage. *D*, Infill cumulative tonnage as a function of cell tonnage uncertainty. *E*, Development histogram of total tonnage. *F*, Development cumulative tonnage as a function of cell tonnage uncertainty.

Aggregation

The last step in the modeling is the aggregation of the resources of individual beds. Errors in different realizations of the same beds are independent because each realization has the same probability of being the true deposit. The same is true for the errors between two realizations of two different beds. For example, errors in the first realization for Anderson are not correlated with the errors in any of the realizations for Roland. Under such circumstances, the aggregation must be done randomly (Schuenemeyer and Gautier, 2010), using one realization per bed, with each realization being used exactly once. Considering that the realizations are prepared in no particular order, the most straightforward way to pair them is in the same order they were generated: add the first realization for the Canyon coal bed to the first realization for Anderson, Smith, and Roland, the second realization for the Canyon coal bed to the second realization of the other three coal beds, and so on. Selected results are shown in figures 32 and 33. In general, percentiles are not additive. For example, the 5th percentile of the total resources is different from the sum of the 5th percentiles of the resources of all coal beds.

The left column of figure 34 is an average of the realizations for the three drilling stages. Not surprisingly, the mean maps are smoother than the realizations, but on average, they tell the same story. Mean maps resemble those obtained by using methods such as kriging or inverse distance weighting. Mining companies, in particular, are more interested in local variability. Hence, realizations are more appropriate for planning purposes. Cell uncertainty for total resources is similar to that for individual coal beds. All in all, most of the cell uncertainty decreases as more data become available, except for some areas with complex geology where denser drilling exposes more complex variability that increases the maximum uncertainty. In our case, the anomalies are in two areas. High spreads in township T. 46 N., R. 73 W. are associated with a channel in the Anderson coal bed, and those along T. 49 N. are associated with localized increase in thickness in the Roland coal bed.

The total resource distributions on the right-hand side of figure 35 show good convergence to 39 bst. A comparison of the cell uncertainty statistics on the left side of figure 35 with previous results, shows that any percentile, say the 40th percentile of the 5−95 spread for reconnaissance—457 kst—is larger than the largest individual 40th percentile—325 kst—but smaller than their sum—184 + 325 + 75 + 79 kst.

Conclusions

National and international standards for reporting uncertainty associated with the assessment of mineral resources and mineable reserves avoid prescribing methods for conducting the modeling. Geostatistics provides an adequate approach to the modeling of uncertainty in coal resources and should be a preferred tool in the assessment of coal or other mineral deposits. The formulations are general enough to accommodate different geological realities, scales of study, and geometry and abundance of drill holes. The basic elements for the modeling of uncertainty are multiple scenarios characterizing the different deposits that are compatible with the data and the style of geographical variability in the resources. These scenarios are valuable subproducts for evaluating and understanding the geology of the coal beds.

We recommend preparation of two measures of uncertainty, one for the total resource and another for the individual cells that make up the total resource. Both measures allow statistical analysis according to universally accepted standards, such as percentiles and confidence intervals. Such analysis is not possible with the traditional distance classification schemes, such as the one described in USGS Circular 891 (Wood and others, 1983). The measures show that (a) commonly the same drilling holes result in different uncertainties for different coal beds, (b) different average degrees in the reduction of uncertainty for the same drilling densification, (c) rates of uncertainty reduction smaller and not proportional to the increase in the number of drill holes, (d) the possibility of having higher uncertainty in areas of more complex geology, (e) sometimes uncertainty is proportional to cell tonnage, and (f) for exactly the same drilling pattern, aggregation of tonnage per bed has different uncertainty than the individual beds. These are all important effects not possible to model with distance methods.

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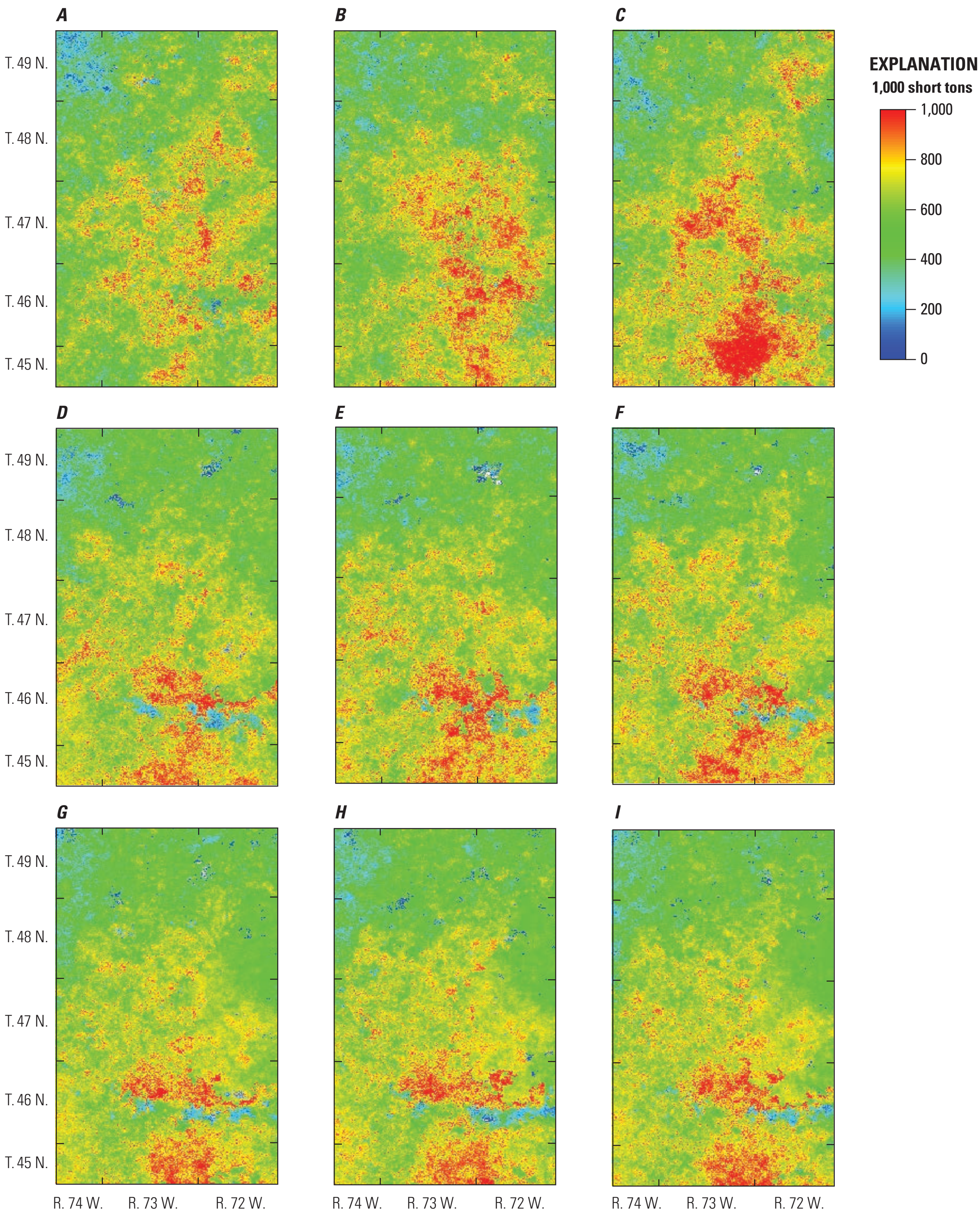


Figure 32. Aggregated tonnage realizations for the four modeled coal beds, selected percentiles. *A*, Reconnaissance 5th percentile. *B*, Reconnaissance median. *C*, Reconnaissance 95th percentile. *D*, Infill 5th percentile. *E*, Infill median. *F*, Infill 95th percentile. *G*, Development 5th percentile. *H*, Development median. *I*, Development 95th percentile.

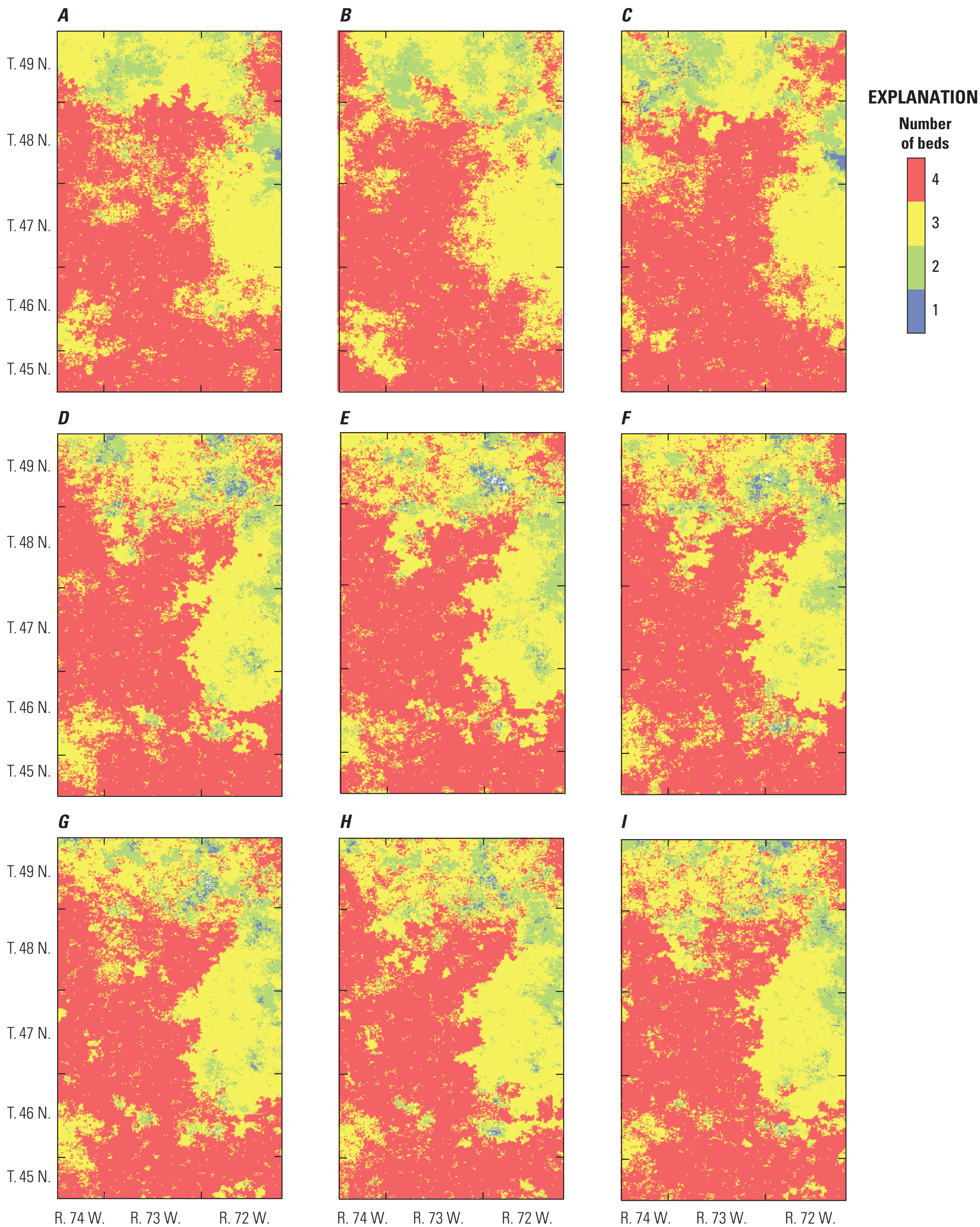


Figure 33. Number of beds contributing to the aggregated tonnage for the corresponding percentiles in the realization shown in figure 32.

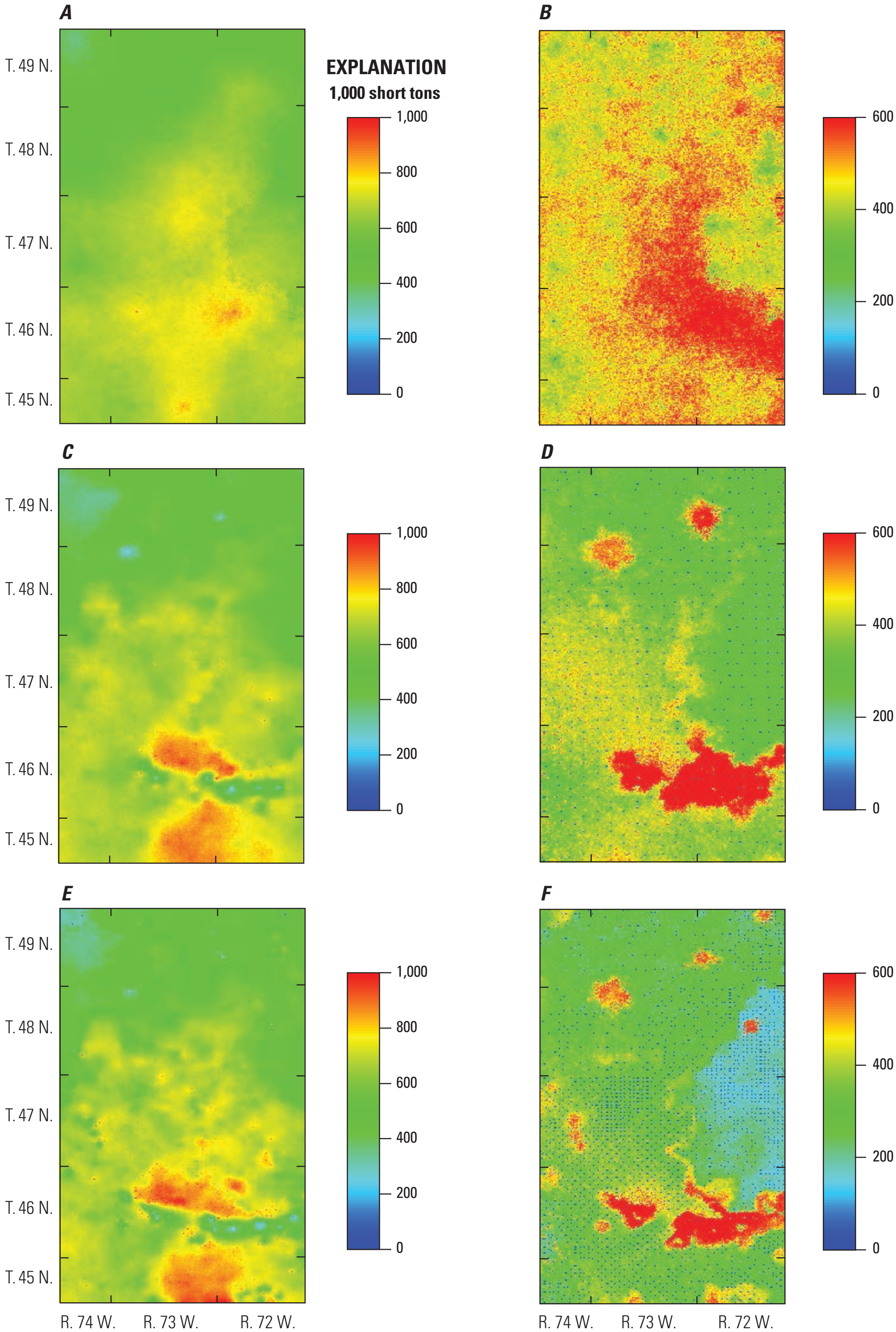


Figure 34. Mapping of two cell statistics for aggregated resources. *A*, Reconnaissance mean tonnage. *B*, Reconnaissance 5–95 spread. *C*, Infill mean tonnage. *D*, Infill 5–95 spread. *E*, Development mean tonnage. *F*, Development 5–95 spread.

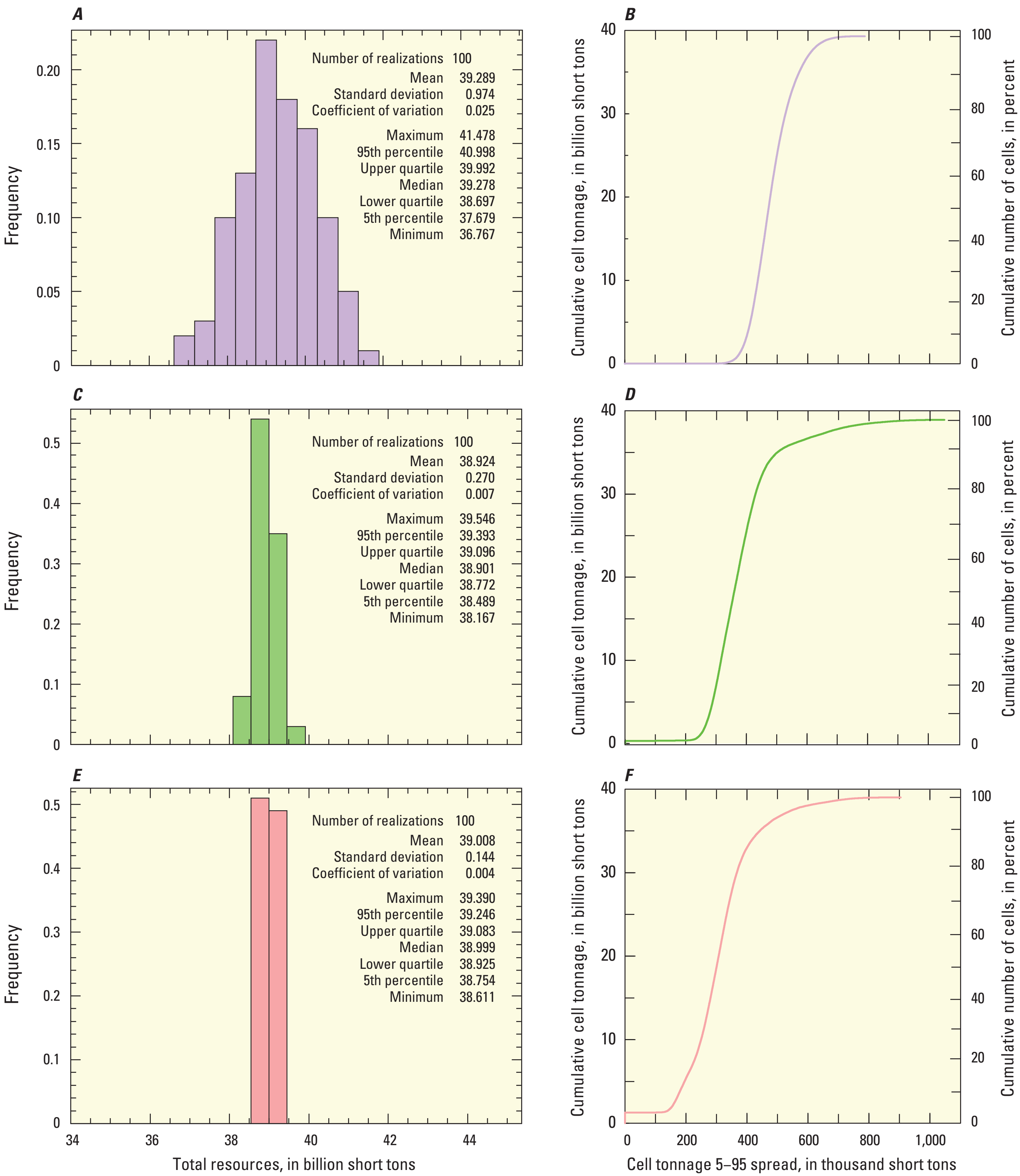


Figure 35. Uncertainty in aggregated resources. *A*, Reconnaissance histogram of total tonnage. *B*, Reconnaissance cumulative tonnage as a function of cell tonnage uncertainty. *C*, Infill histogram of total tonnage. *D*, Infill cumulative tonnage as a function of cell tonnage uncertainty. *E*, Development histogram of total tonnage. *F*, Development cumulative tonnage as a function of cell tonnage uncertainty.

References Cited

Caumon, G., Strebelle, S., Caers, J.K. and Journel, A.G., 2004, Assessment of global uncertainty for early appraisal of hydrocarbon fields: Society of Petroleum Engineers paper 89943, 8 p.

Chaparro, L.F., 2011, Signals and systems using Matlab: Burlington, Mass., Academic Press, 752 p.

Chilès, J.P., and Delfiner, Pierre, 2012, Geostatistics—Modeling spatial uncertainty (2d ed.): Hoboken, N.J., John Wiley & Sons, 699 p.

CRIRSCO (Combined Reserves International Reporting Standards Committee), 2013, International reporting template for the public reporting of exploration results, mineral resources and mineral reserves, 41 p., accessed September 2014 at *http://www.crirsco.com/crirsco_template_may2013.pdf*.

de Bono, E., 1970, Lateral thinking—Creativity step by step: New York, Harper & Row, 300 p.

de Souza L.E., Costa, J.L.C.L., and Koppe, J.C., 2004, Uncertainty estimate in resource assessment—A geostatistical contribution: Natural Resources Research, v. 13, no. 1, p. 1–15.

Deutsch, C.V., 2006, A sequential indicator simulation program for categorical variables with point and block data—BlockSIS: Computers & Geosciences, v. 32, no. 10, p. 1669–1681.

Deutsch, C.V., and Journel, A.G., 1998, GSLIB—Geostatistical software library and user's guide (2d ed.): New York, Oxford University Press, 369 p. and 1 CD.

Hengl, Tomislav, 2006, Finding the right pixel size: Computers & Geosciences, v. 32, no. 9, p.1283–1298.

Hengl, Tomislav, 2007, A practical guide to geostatistical mapping of environmental variables: Ispra, Italy, Joint Research Centre, Institute for Environment and Sustainability, 143 p., also accessed September 2014 at *http://eusoils.jrc.ec.europa.eu/esdb_archive/eusoils_docs/other/EUR22904en.pdf*.

Hohn, M.E., and Britton, J.Q., 2013, A geostatistical case study in West Virginia—All coals are not the same: International Journal of Coal Geology, v. 112, p. 125−133.

Geovariances, 2014, Isatis: Accessed September 2014 at *http://www.geovariances.com/en/isatis-ru324*.

Jones, T.A., Hamilton, D.E., and Johnson, C.R., 1986, Contouring geologic surfaces with the computer: New York, Van Nostrand Reinhold, 314 p.

JORC (Joint Ore Reserves Committee), 2004, Australasian code for reporting of exploration results, mineral resources and ore reserves: Accessed September 2014 at *http://www.jorc.org/docs/jorc2004web_v2.pdf*.

JORC (Joint Ore Reserves Committee), 2012, Australasian code for reporting of exploration results, mineral resources and ore reserves: Accessed September 2014 at *http://www.jorc.org/docs/jorc_code2012.pdf*.

Journel, A.G., and Kyriakidis, P.C., 2004, Evaluation of mineral reserves—A simulation approach: New York, Oxford University Press, 216 p.

Krige, D.G., 1951, A statistical approach to some basic mine valuation problems on the Witwatersrand: Journal of the Chemical, Metallurgical and Mining Society of South Africa, v. 52, no. 6, p. 119−139.

Luppens, J.A., Scott, D.C., Haacke, J.E., Osmonson, L.M., Rohrbacher, T.J., and Ellis, M.S., 2008, Assessment of coal geology, resources, and reserves in the Gillette coalfield, Powder River Basin, Wyoming: U.S. Geological Survey Open-File Report 2008–1202, 127 p., accessed September 2014 at *http://pubs.usgs.gov/of/2008/1202/*.

Luppens, J.A., Wilson, S.E., and Stanton, R.W., eds.,1992, Manual on drilling, sampling and analysis of coal: Philadelphia, Pa., American Society for Testing and Materials, ASTM Manual Series, MNL 11, 61 p.

Manchuk, J.G., Leuangthong, O., and Deutsch, C.V., 2009, The proportional effect: Mathematical Geosciences, v. 41, no. 7, p. 799–816.

Matheron, Georges, 1963, Principles of geostatistics: Economic Geology, v. 58, p. 1246−1266.

Njowa, Godknows, Clay, A.N., and Musingwini, Cuthbert, 2014, A perspective on global harmonization of major national mineral asset valuation codes: Resources Policy, v. 39, p. 1−14.

Olea, R.A., 2009, A practical primer on geostatistics: U.S. Geological Survey Open-File Report 2009–1103, 346 p., accessed September 2014 at *http://pubs.usgs.gov/of/2009/1103*.

Olea, R.A., and Luppens, J.A., 2012, Sequential simulation approach to modeling of multi-seam coal deposits with an application to the assessment of a Louisiana lignite: Natural Resources Research, v. 21, no. 4, p. 443−459.

Olea, R.A., Luppens, J.A., and Tewalt, S.J., 2011, Methodology for quantifying uncertainty in coal assessments with an application to a Texas lignite deposit: International Journal of Coal Geology, v. 85, no. 1, p. 78–90.

Pyrcz, M.J., and Deutsch, C.V., 2014, Geostatistical reservoir modeling (2d ed.): New York, Oxford University Press, 433 p.

Remy, Nicolas, Boucher, Alexander, and Wu, Jianbing, 2009, Applied geostatistics with SGeMS—A user's guide: Cambridge, UK, Cambridge University Press, 264 p. and 1 CD.

Rendu, J.-M., 2006, Reporting mineral resources and mineral reserves in the United States of America—Technical and regulatory issues, *in* Proceedings, Sixth International Mining Geology Conference: Australasian Institute of Mining and Metallurgy, p. 11−20.

Rescher, Nicholas, 2007, Error—On our predicament when things go wrong: Pittsburgh, Pa., University of Pittsburgh Press, 116 p.

Ribeiro, P.J., Jr., and Diggle, P.J. [n.d.], geoR—A package for geostatistical data analysis using the R software: Accessed September 2014 at *http://www.leg.ufpr.br/geoR/*.

Schuenemeyer, J.H., and Gautier, D.L., 2010, Aggregation methodology for the Circum-Arctic resource appraisal: Mathematical Geosciences, v. 42, no. 5, p. 583–594.

Scott, D.C., and Luppens, J.A., 2012, Assessment of coal geology, resources, and reserve base in the Powder River Basin, Wyoming and Montana: U.S. Geological Survey Fact Sheet 2012–3143, 6 p., accessed September 2014 *http://pubs.er.usgs.gov/publication/fs20123143*.

SEC (Securities and Exchange Commission) [n.d.], Industry Guide 7—Description of property by issuers engaged or to be engaged in significant mining operations. Accessed September 2014 at *http://web.cim.org/standards/documents/Block474_Doc32.pdf*.

Solow, A.R., 1985, Bootstrapping correlated data: Mathematical Geology, v. 17, no. 7, p. 769–775.

Srivastava, M.R., 2013, Geostatistics—A toolkit for data analysis, spatial prediction and risk management in the coal industry: International Journal of Coal Geology, v. 112, p. 2–13.

Wood, G.H., Jr., Kehn, T.M., Carter, M.D., and Culbertson, W.C., 1983, Coal resources classification system of the U.S. Geological Survey: U.S. Geological Survey Circular 891, 65 p.

Zellner, Arnold, Keuzenkamp, H.A., and McAleer, Michael, eds., 2001, Simplicity, inference and modelling—Keeping it sophisticatedly simple: Cambridge, UK, Cambridge University Press, 302 p.

Glossary

First occurrence of the term in the report appears in italics. Some terms have additional meanings, but the glossary includes only those used in this report.

A

aliasing Distorted modeling of anomalies smaller than twice the drilling spacing.

average Mean.

C

coefficient of variation The ratio of the standard deviation over the mean.

confidence interval A range of values calculated from sample observations and supposed to contain the true parameter value with a given probability.

correlation Interdependence between two variables. When linear, it is measured by a coefficient that is −1 for perfect negative, or inverse, correlation; 0 in the absence of any correlation, and +1 for perfect positive correlation.

coverage The proportion of times a true value falls inside a confidence interval.

cumulative distribution function A mathematical expression providing the probability that the value of a random variable is less than any given value.

E

estimation The process of providing a numerical value for an unknown quantity based on the information provided by a sample.

G

geostatistics A branch of statistics in which all inferences are made by taking into account the style of spatial fluctuation of the variables and the location of each observation.

H

histogram A graphical display of an empirical probability distribution. The values of the random variable are divided into multiple intervals called bins; all values are allocated to the bins; final relative counts are displayed as bars.

K

kriging A group of geostatistical estimation methods formulated to minimize estimation errors in a minimum mean square error sense.

L

lower quartile In a split of a ranked sample into four parts of equal size, the divider between the two partitions below the median. It is synonymous with the 25th percentile.

M

mean A measure of centrality in a sample, population or probability distribution. For a sample, the mean is equal to the sum of all values divided by the sample size,

$$\bar{z} = \frac{1}{n}\sum_{i=1}^{n} z_i \, .$$

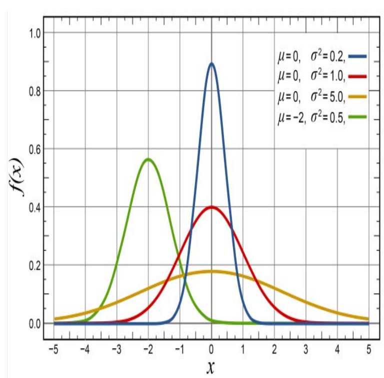
median In a probability distribution or ranked sample or population, the divider evenly splitting the observations into two halves of equal size: a half of lowest values and a half of highest values. It is a measure of centrality and is synonymous with the 50th percentile.

N

normal distribution The family of symmetric, bell-shaped functions that indicates the probability, $f(x)$, that the random variable will be between any two values of x:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\left(\frac{x-\mu}{\sigma} \right)^2 \right]$$

where μ is the mean and σ is the standard deviation of the probability density function.



Nyquist sampling theorem A fundamental statement of information theory declaring the impossibility of properly reconstructing anomalies which are smaller than twice the sampling interval.

P

parsimony Simplicity in a mathematical model, particularly with respect to the minimum number of parameters.

percentile In a probability distribution, sample, or population sorted by increasing observation value, each one of the 99 dividers that produce exactly 100 subsets with equal number of observations. The dividers are sequential ordinal numbers starting from the one between the two groups with the lowest values. The dividers are used to denote the proportion of values above and below them.

population The complete set of all specimens comprising a system of interest and from which data can be collected. For the tonnage of a deposit, the population is any complete set of weight measurements that could be taken, adding to the deposit weight.

probability A measure of the likelihood of occurrence of an event. It takes real values between 0 and 1, with 0 denoting absolute impossibility and 1 total certitude. Sometimes probabilities are multiplied by 100 to express them as percentages.

probability density function An analytical expression, $f(x)$, describing the relative likelihood of a random variable. For discrete random variables, $f(x)$ directly provides the likelihood of each value of the variable; for a continuous random variable, the area under $f(x)$ between any two values of the random variable provides the likelihood of the interval. The likelihood of the random variable taking any value less than a specified value is the cumulative distribution function.

probability distribution Probability density function.

proportional effect Dependency between the variability of the cell mean and any measure of cell uncertainty, such as the standard error or a confidence interval.

Q

quartile In a distribution or ranked sample or population, any of the three dividers that separates the observations in four parts of equal size.

R

random function A collection of random variables.

random variable The collection of all possible outcomes in an event or study, and their associated probability of occurrence.

realization Any of the infinite outcomes of a random function.

S

sample (a) In geology, a specimen taken for inspection, analysis, or display. (b) In statistics, a representative subset of a population comprising observations for several specimens.

sample size The number of observations in a dataset.

standard deviation The square root of the variance.

standard error The standard deviation of the probability distribution of an estimate.

stochastic simulation Mathematical modeling of a complex system using probabilistic methods involving random variables.

T

tessellation Subdivision of a plane into one or more geometric shapes without gaps and overlaps.

U

upper quartile In a split of a sample into four parts of equal size, the divider between the two partitions above the median. It is equivalent to the 75th percentile.

V

variance A measure of spread in a sample, population or probability distribution. For a sample, it is equal to the sum of the square of all observations minus the mean divided by the sample size minus 1,

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(z_i - \bar{z}\right)^2 .$$

Toolkit for Generating Coal Deposit Realizations by Stochastic Simulation

A realization is analogous to drawing a hand from a deck of cards. Realizations are specific outcomes out of a large to infinite number of possibilities. Here, we restrict our attention to realizations of two-dimensional attributes, such as thickness of a coal bed or tonnage of a coal deposit, which can always be displayed as pixel maps. Generation of realistic realizations taking maximum advantage of any of the methods available in geostatistics may require application of more than one method depending on the complexity of the geology and the location of the available data. This toolkit discusses the procedures required to generate the realizations necessary to model uncertainty for the Gillette coal field and a few other situations of interest. In practice, generation of realizations requires the use of some of the numerous available computer software packages, such as GSLIB (Deutsch and Journel, 1998), SGeMS (Remy and others, 2009), Isatis (Geovariances, 2014), or geoR (Ribeiro and Diggle [n.d.]). We used SGeMS for the calculations and GSLIB for the displays.

The modeling is based on the assumption that thickness data are always available. These data are supposed to be free of institutional errors, such as mistakes in reporting location, inconsistencies in picking the top and bottom of each coal bed, or partial penetration of a bed by a drill hole (Luppens and others, 1992). Location coordinates must be in some Cartesian system, such as Universal Transverse Mercator.

The first step in the generation of any realization is the definition of the study area and resolution of the realizations. Most geostatistical software allows generation of realizations inside a rectangular area with sides parallel to the principal axes of the coordinate system. Under such limitations, it is often necessary either to truncate the zone of interest or to extend the rectangle beyond the expected or known boundaries of a deposit.

Although coal deposits are partly continuous, modeling realizations requires discretization of the area inside the rectangle. Values are estimated only at selected locations, usually at the nodes of a square grid. The nodes can also be regarded as centers of square cells tessellating the study area. Modelers have some freedom in selecting the cell size as long as they are guided by the following common sense considerations (Jones and others, 1986; Hengl, 2006, 2007; Pyrcz and Deutsch, 2014): (a) the average sampling spacing is always a good starting reference to decide a cell size; (b) the larger the cell size, the lower the processing time and size of the generated files, but the tessellations must be such that no cell contains more than one data value or, at most, only a small fraction do so; (c) at the other extreme, the cell size should not be smaller than the magnitude of the smallest detail considered worth modeling. Given a sufficiently high resolution, the discretization will be unnoticeable in a map.

Results based on a group of coal bed realizations tend to stabilize after 40–80 realizations (for example, de Souza and others, 2004). Results vary with the geology of the deposit, the sampling, and the statistics of interest. The only conclusive way to explore the convergence of results is through sensitivity analysis. The standard practice is to assume that 100 realizations are sufficient.

When performing grid-to-grid operations among realizations for different coal beds, the pairing should be random. Considering that each realization is equally likely to be the real surface, no special care is required to ensure randomness when realizations for different coal beds are generated using different random seeds. A simple pairing in the same order in which the different sets of realizations have been generated is always adequate. Some operations require preparation of ad hoc utility programs to supplements the capabilities provided by the standard geostatistical packages.

Initial Assumptions

For the purpose of considering different situations of interest, the following assumptions are made for the simplest case, which serves as the basic scenario upon which more elaborate scenarios and modeling procedures are built:

1. The deposit is deep enough that no oxidation has taken place.
2. There are no drill holes completely missing the coal bed(s).
3. Thickness data are evenly distributed throughout the study area so that modeling is possible everywhere.
4. Constant density is necessary to assume either for lack of data or because it is sufficient for the modeling.
5. There is only one bed or the data have been aggregated into one value per drill hole.
6. There are no faults with significant displacements in the study area.

Procedure A. Basic scenario

Given the assumptions stated in the preceding section, the following is satisfactory for generating the tonnage realizations:

Step 1. Define the boundaries of the study area and cell size.

Step 2. Generate at least 100 realizations of thickness.

Step 3. Convert the thickness realizations to tonnage realizations by multiplying all cell values by the appropriate conversion factor.

Although this procedure is straightforward, deposits of practical interest rarely follow all the assumptions in this simple scenario. Other procedures are given below for addressing various other possibilities of interest.

Procedure B. Basic scenario, except for constant depth oxidation

Assumption 1 is no longer true, but Assumptions 2–6 still hold. In that situation, do the following:

Step 1. Define the boundaries of the study area and cell size.

Step 2. Prepare a digital elevation map of land surface with readings at the same cells as in the assessment grid.

Step 3. Generate at least 100 realizations for the elevation of the roof of the coal bed.

Step 4. Compare realizations for the roof elevation to the land surface elevation grid to prepare oxidation indicator grids. For each realization, j, and cell location, $\mathbf{u}_i$, calculate the oxidation indicator, $O_j(\mathbf{u}_i)$:

$$O_j\left(\mathbf{u}_i\right)=\begin{cases}0, \text{ if } surface-oxidationdepth \le roof \\ 1, \text{ otherwise}\end{cases}$$

Step 5. Generate the same number of thickness realizations used in Step 3.

Step 6. Pairing oxidation and thickness realizations, blank all cells where the oxidation indicator is 0.

Step 7. Convert the thickness realizations to tonnage realizations by multiplying all non-blank cell values by the appropriate conversion factor.

Procedure C. Basic scenario, except for variable depth oxidation

Assumption 1 in the basic scenario is no longer true, but Assumptions 2–6 are still valid. Modeling of variable depth of oxidation requires the availability of oxidation depth data from most drill holes. Otherwise, the modeler has to assume constant oxidation depth. If sufficient oxidation depth data are available, then:

Step 1. Define the boundaries of the study area and cell size.

Step 2. Generate at least 100 realizations for the elevation of the base of the oxidized zone.

Step 3. Generate the same number of elevation realizations for the roof of the coal bed.

Step 4. Compare the two sets of realizations. For all cell locations, $\mathbf{u}_i$, for a pair of realizations, j, for base and roof, calculate the oxidation indicator, $O_j(\mathbf{u}_i)$:

$$O_j\left(\mathbf{u}_i\right)=\begin{cases}0, \text{ if } oxidationbase \le coalroof \\ 1, \text{ otherwise}\end{cases}$$

Step 5. Generate the same number of thickness realizations as used in Steps 2 and 3.

Step 6. Pairing oxidation and thickness realizations, blank all cells for which the oxidation indicator is 0.

Step 7. Convert the thickness realizations to tonnage realizations by multiplying all non-blank cell values by the appropriate conversion factor.

Procedure D. Basic scenario, except that the deposit does not continuously extend over the study area

Coal deposits rarely extend with perfect continuity over the entire study area. Areas where coal is absent are revealed by drill holes not penetrating the coal bed after reaching what should be sufficient depth. Violation of Assumption 2 requires the following modifications to the basic procedure:

Step 1. Define the boundaries of the study area and the cell size.

Step 2. Use the thickness data to generate a second dataset of transformed values, $I(\mathbf{s}_i)$, to denote only presence or absence of the bed of interest. For each observation, calculate:

$$I\left(\mathbf{s}_i\right)=\begin{cases}0, \text{ if } thickness = 0 \\ 1, \text{ otherwise}\end{cases}$$

Step 3. Generate at least 100 realizations for the indicator data $I(\mathbf{s}_i)$ by using sequential indicator simulation (see, for example, Deutsch, 2006).

Step 4. Generate the same number of thickness realizations as for the presence-absence indicators in Step 3.

Step 5. Pair the presence-absence indicator and thickness realizations and blank those cells for which the presence-absence indicator is 0.

Step 6. Convert thickness realizations to tonnage realizations by multiplying all non-blank cell values by the appropriate factor.

Procedure E. Basic scenario, except that data are missing in at least one large part of the study area

The north-south orientation of a rectangular study area often does not match the orientation and shape of the deposit. Such mismatches may result in areas without data, a violation of Assumption 3. Geostatistical methods have limited extrapolation power. Beyond a critical distance, the validity of extrapolation depends on the spatial continuity of the deposit; hence, it is better to avoid any modeling. These areas to be completely ignored in the modeling lie in what is roughly equivalent to the geographical extension of the hypothetical resources in other approaches, such as the one of Wood and others (1983). The following procedure is suited to addressing violation of Assumption 3:

Step 1. Define the boundaries of the study area and cell size.

Step 2. Prepare a second dataset, $I(\mathbf{s}_i)$, indicating only presence or absence of the deposit at each drill hole location, $\mathbf{s}_i$:

$$I(\mathbf{s}_i) = \begin{cases} 0, \text{ if } \textit{thickness} = 0 \\ 1, \text{ otherwise} \end{cases}$$

Step 3. Krige the presence-absence indicators and prepare a standard error map.

Step 4. Generate at least 100 realizations of thickness.

Step 5. Blank all cells where the standard error is greater than a critical threshold. Conceptually this blanking is similar to limiting the estimation to the zone of influence of a drill hole. A good default value for the threshold is 0.5. Sensitivity analysis should be used to justify other values.

Step 6. Convert the thickness realizations to tonnage realizations by multiplying all non-blank cell values by the appropriate conversion factor.

Procedure F. Basic scenario, except that density data are sufficient for generating density realizations

In this situation Assumption 4 does not hold, in which case an adequate procedure is the following:

Step 1. Define the boundaries of the study area and cell size.

Step 2. Generate at least 100 thickness realizations.

Step 3. Generate an equal number of density realizations.

Step 4. Pairing the thickness and density realizations at random, perform a grid-to-grid multiplication of all nodal values cell by cell.

Procedure G. Basic scenario, except that the deposit needs to be modeled bed by bed

When aggregation of the data at each drill hole is not acceptable because modeling of each individual coal bed is a requirement, each individual coal bed must be modeled by using the appropriate procedure for single-bed modeling. The last step in the procedure is the aggregation of resources in each coal bed individually modeled, which is accomplished by pairing one realization of each of the beds and performing a grid-to-grid operation of summing the cell values, ignoring all blank cells.

Procedure H. Basic scenario, except that the deposit is crossed by a few faults

If the faults are few and break the deposit into large blocks, model each block separately. If faults are numerous and break the deposit into many small blocks, the deposit would not likely be of economic interest; hence, no modeling procedure is offered for that situation.

Procedure I. Basic scenario, except that the deposit is neither continuous nor to be modeled by aggregating the drill hole data for multiple coal beds

The valid assumptions are now the following:

1. The deposit is deep enough so that no oxidation has taken place.

2. There are no large parts of the study area without drill holes.

3. Constant density is either necessary to assume either for lack of data or because it is sufficient for the modeling.

4. There are no faults with significant displacements in the study area.

In this situation, do the following:

Step 1. Define the boundaries of the study area and cell size.

Step 2. For every coal bed:

Substep A. Generate a second dataset of indicators, $I(\mathbf{s}_i)$, denoting whether the bed is present or absent at each drill hole location, $\mathbf{s}_i$:

$$I(\mathbf{s}_i) = \begin{cases} 0, \text{ if } \textit{thickness} = 0 \\ 1, \text{ otherwise} \end{cases}$$

Substep B. Generate at least 100 presence-absence indicator realizations.

Substep C. Generate the same number of thickness realizations.

Substep D. Pair the presence-absence indicator and thickness realizations and blank those cells for which the presence-absence indicator is 0.

Substep E. Convert the thickness realizations to tonnage realizations by multiplying all non-blank cell values by the appropriate conversion factor.

Step 3. Aggregate tonnage realizations bed by bed ignoring all blank cells, by paring one of each bed realizations and then adding collocated cells.

Procedure J. Scenario of oxidized and discontinuous deposit with multiple beds requiring individual modeling

The only valid simplifications are the following:

1. There are sufficient thickness data evenly scattered across the study area.
2. There are no density data, so constant density is assumed.
3. There are no faults.

In addition, because there are very few oxidation depth measurements, oxidation depth will be assumed and set equal to the average of the few measurements available. In this situation, the following steps are recommended:

Step 1. Define the boundaries of the study area and cell size.

Step 2. Prepare a digital elevation grid.

Step 3. For every coal bed:

Substep A. Generate at least 100 realizations for the roof of the coal bed.

Substep B. Generate the same number of oxidation realizations, $O_j(\mathbf{u}_i)$, by comparing for every cell, $\mathbf{u}_i$, roof elevation and oxidation base given by surface elevation minus the fixed oxidation depth:

$$O_j\left(\mathbf{u}_i\right) = \begin{cases} 0, \text{ if } \mathit{surface} - \mathit{oxidationdepth} \le \mathit{roof} \\ 1, \text{ otherwise} \end{cases}$$

Substep C. Generate a second dataset, $I(\mathbf{s}_i)$, denoting presence or absence of the coal bed at every drill hole location, $\mathbf{s}_i$:

$$I\left(\mathbf{s}_i\right) = \begin{cases} 0, \text{ if } \mathit{thickness} = 0 \\ 1, \text{ otherwise} \end{cases}$$

Substep D. Generate as many presence-absence indicator realizations as roof realizations in Substep A.

Substep E. Pairing oxidation and presence-absence indicator realizations, blank all oxidation realization cells for which the presence-absence indicator cell is 0.

Substep F. Using the thickness data, produce as many thickness realizations as in Substep A.

Substep G. Pairing the thickness and oxidation realizations, blank all thickness cells for which the collocated oxidation cell is already blank.

Substep H. Convert the thickness realizations to tonnage realizations by multiplying all non-blank cell values by the appropriate factor.

Step 4. Pair tonnage realizations, one from every bed, and add collocated cell values, ignoring blank cells.

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