Prepared in cooperation with the St. Croix Watershed Research Station

Development of Regression Equations to Revise Estimates of Historical Streamflows for the St. Croix River at Stillwater, Minnesota (Water Years 1910–2011), and Prescott, Wisconsin (Water Years 1910–2007)

Scientific Investigations Report 2014–5239

U.S. Department of the Interior
U.S. Geological Survey

Cover photograph: The lift bridge at Stillwater, Minnesota is raised up, and a riverboat is passing underneath it. The solar panel, wire-weight gage, antenna, and box housing the Data Collection Platform are shown in the left-center of the photo, attached to the bridge. Photograph taken on June 2, 2014, by Brett Savage, U.S. Geological Survey, Minnesota Water Science Center.

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U.S. Geological Survey

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Conversion Factors

Inch/Pound to International System of Units

Multiply	By	To obtain
	Length	
foot (ft)	0.3048	meter (m)
mile (mi)	1.609	kilometer (km)
	Area	
square mile (mi^2)	2.590	square kilometer (km^2)
	Flow rate	
cubic foot per second (ft^3/s)	0.02832	cubic meter per second (m^3/s)
	Mass	
ton, short (2,000 pounds)	0.9072	megagram (Mg)

Horizontal coordinate information is referenced to the North American Datum of 1983 (NAD 83).

A water year is the 12-month period, October 1 through September 30, and is designated by the calendar year in which it ends.

Abbreviations

ADVM	acoustic Doppler velocity meter
ANOVA	analysis of variance
MCES	Metropolitan Council Environmental Services
NSE	Nash-Sutcliffe Efficiency
NWIS	National Water Information System [U.S. Geological Survey database]
R^2	coefficient of determination
TMDL	total maximum daily load
USACE	U.S. Army Corps of Engineers
USGS	U.S. Geological Survey

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By Jeffrey R. Ziegeweid and Suzanne Magdalene

Abstract

A natural dam of glacial-era sediments at the confluence of the St. Croix and Mississippi Rivers forms Lake St. Croix, a riverine lake that comprises the lowest 25 miles of the St. Croix River. Historically, backwater effects from the Mississippi River prevented the use of traditional streamgages for collecting continuous streamflow data needed to calculate nutrient loads at the inlet to and outlet from Lake St. Croix at Stillwater, Minnesota and Prescott, Wisconsin, respectively. The development of index-velocity streamgages has enabled the measurement of continuous streamflow in backwater conditions using continuously measured velocities at the streamgage. Index-velocity streamgages were installed at Prescott, Wisconsin, and Stillwater, Minnesota, in 2007 and 2011, respectively.

Continuous daily mean streamflow data from the new index-velocity streamgages, long-term upstream streamgages, and tributary streamgages were used to (1) develop regression equations that improve estimates of historical streamflow at Stillwater and Prescott, (2) evaluate the accuracies of new and previous equations used to estimate historical streamflows, and (3) compute and evaluate revised estimates of historical streamflows for Stillwater for water years 1910–2011 and for Prescott for water years 1910–2007. The abilities of previous and newly developed regression equations to accurately estimate streamflows were evaluated using Nash-Sutcliffe Efficiency (NSE) values. The NSE values at Stillwater improved from 0.90 to 0.98, and the NSE values at Prescott improved from 0.77 to 0.94.

The new regression equations were used to calculate revised estimates of historical streamflows for Stillwater and Prescott starting in 1910 and ending when index-velocity streamgages were installed. Monthly, annual, 30-year, and period of record statistics were examined between previous and revised estimates of historical streamflows. The abilities of the new regression equations to estimate historical streamflows were evaluated by using percent differences to compare new estimates of historical daily streamflows to discrete streamflow measurements made at Stillwater and Prescott before the installation of index-velocity streamgages. Although less variability was observed between estimated and measured streamflows at Stillwater compared to Prescott, the percent difference data indicated that the new estimates closely approximated measured streamflows at both locations.

Introduction

A natural dam of glacial-era sediments at the confluence of the St. Croix and Mississippi Rivers creates Lake St. Croix, a riverine lake that comprises the lowest 25 miles (mi) of the St. Croix River along the border between Minnesota and Wisconsin (fig. 1; Triplett and others, 2009). Increased eutrophication in Lake St. Croix throughout recent decades is linked to increased agriculture and urban development in the St. Croix Basin (Minnesota Pollution Control Agency and Wisconsin Department of Natural Resources, 2012). Concerns about future water quality in the St. Croix River (including Lake St. Croix) prompted several agencies and organizations to form the St. Croix Basin Water Resources Planning Team in 1993 to coordinate research and monitoring efforts in the St. Croix Basin and address the issue of increased eutrophication (Magdalene and others, 2013). Monitoring efforts have included nutrient monitoring at the inlet to and outlet from Lake St. Croix by Metropolitan Council Environmental Services (MCES) since 1976. However, subsequent nutrient-loading analyses required streamflow data at the inlet to and outlet from Lake St. Croix at Stillwater, Minnesota, and Prescott, Wisconsin, respectively (hereafter referred to as Stillwater and Prescott, respectively; fig. 1).

Traditional streamgages could not be used to measure continuous streamflow at the inlet to and outlet from Lake St. Croix because backwater effects caused by the confluence with the larger Mississippi River confounded the relation between stage and discharge typically used to generate continuous streamflow records. In order to estimate nutrient loads entering and leaving Lake St. Croix, streamflows for the inlet

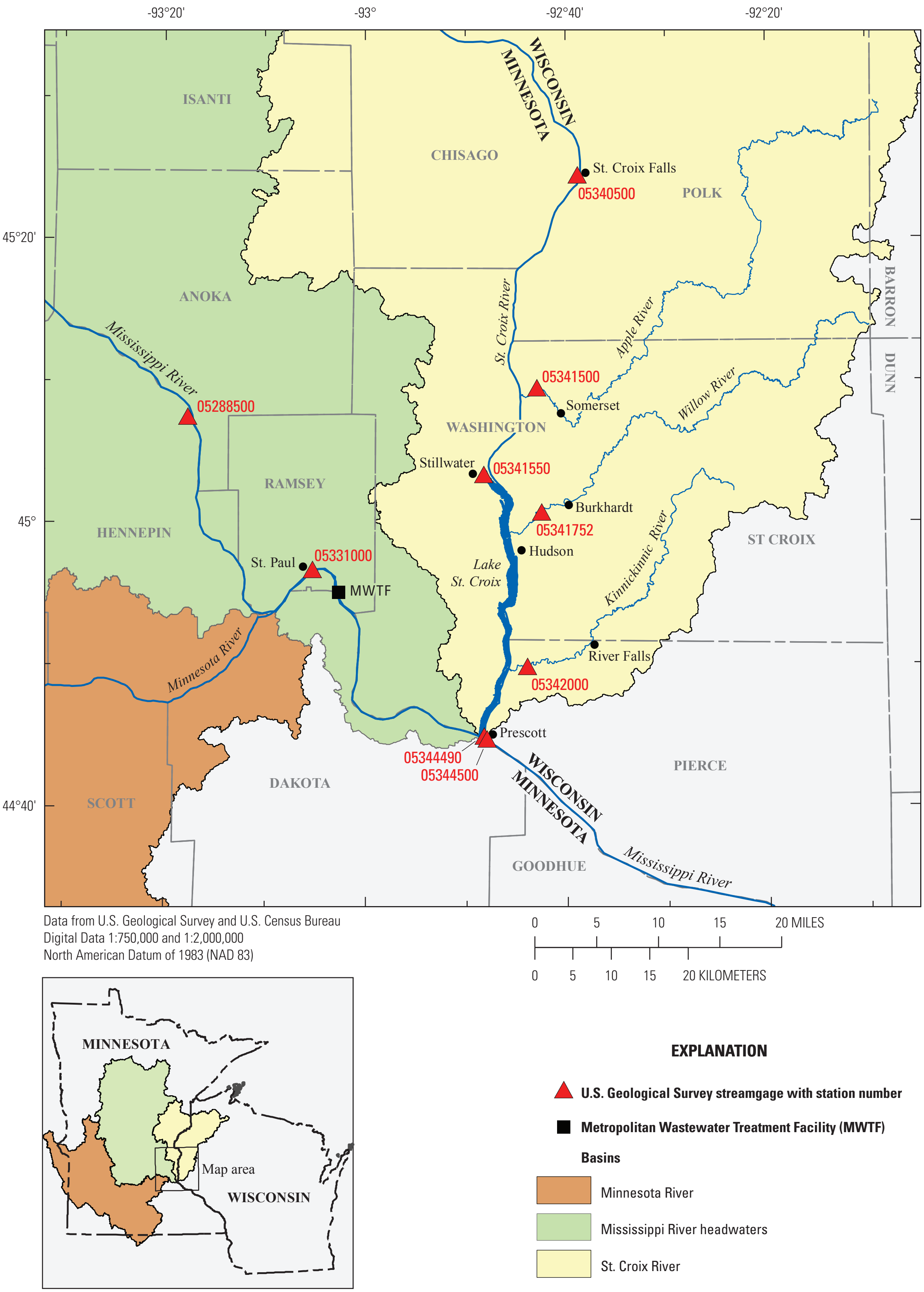


Figure 1. Study area for the St. Croix River in Minnesota and Wisconsin.

to and outlet from Lake St. Croix previously were estimated by combining measured daily streamflows from upstream and tributary streamgages not affected by backwater conditions (LaFrancois and others, 2009; Magdalene and others, 2013). However, the accuracies of these estimates of historical streamflows for the St. Croix River at Stillwater and Prescott were not determined.

In the late 1990s, advances in velocity meters enabled researchers with the U.S. Geological Survey (USGS) to develop index-velocity streamgages to measure continuous streamflow in backwater conditions using continuously measured velocities at the streamgages (Levesque and Oberg, 2012). To obtain more accurate streamflow data for calculating nutrient loads for Lake St. Croix, index-velocity streamgages were installed at Prescott (USGS streamgage 05344490; St. Croix River at Prescott, Wisc.) in 2007 and Stillwater (USGS streamgage 05341550; St. Croix River at Stillwater, Minn.) in 2011 (fig. 1). The new index-velocity streamgages account for backwater effects and provide accurate continuous streamflow data that has improved loading estimates calculated from nutrient data collected by MCES.

The St. Croix Basin Water Resources Planning Team has developed a total maximum daily load (TMDL) for phosphorus in Lake St. Croix, with the goal of eliminating 100 tons in annual phosphorus loads by the year 2020 (Minnesota Pollution Control Agency and Wisconsin Department of Natural Resources, 2012). Improved historical streamflow estimates are needed to evaluate achievement of the TMDL-based phosphorus-reduction goal for Lake St. Croix. Improved estimates of daily streamflow from the index-velocity streamgages can be used in conjunction with streamflow values from upstream and tributary streamgages to develop new regression equations to improve historical streamflow estimates for the St. Croix River at Stillwater and Prescott and to determine the accuracy of historical nutrient-loading estimates at the inlet to and outlet from Lake St. Croix. To address these needs, the U.S. Geological Survey, in cooperation with the St. Croix Watershed Research Station, (1) developed new regression equations to improve historical streamflow estimates at Stillwater and Prescott, (2) evaluated the accuracies of new and previous equations used to estimate historical streamflows, (3) computed revised estimates of historical streamflows for Stillwater and Prescott, and (4) compared previous and revised estimates of historical streamflows for water years 1910–2011 at Stillwater and for water years 1910–2007 at Prescott; a water year is the 12-month period, October 1 through September 30, and is designated by the calendar year in which it ends. This effort built upon a previous study completed by the St. Croix Watershed Research Station, USGS, and MCES (Magdalene and others, 2013), which was funded by the St. Croix River Association.

Purpose and Scope

The purposes of this report are to (1) describe the development of regression equations to estimate historical streamflows of the St. Croix River at Stillwater and Prescott, (2) describe the evaluation of the accuracies of new and previous equations used to estimate historical streamflows, and (3) present revised estimates and evaluations of estimates of historical streamflows for the St. Croix River at Stillwater, Minn. (water years 1910–2011), and at Prescott, Wisc. (water years 1910–2007).

The study area is limited to the lower St. Croix River from St. Croix Falls, Wisconsin to the confluence with the Mississippi River at Prescott, Wisconsin (fig. 1). However, similar techniques could be used to improve discharge estimates in other basins with natural reservoirs of similar slopes, underlying geologies, and hydrologic conditions. Because of issues with serial correlation, the regression analyses used to develop new equations for estimating historical daily mean streamflows should not be used to make inferences about sample variances. However, the developed regression equations are valid for estimating streamflows at Stillwater and Prescott from measured streamflow at St. Croix Falls. In addition, the equations developed to estimate streamflows at Stillwater and Prescott from measured streamflow at St. Croix Falls should only be considered valid for the range of flows observed in the records used to generate these equations. For the rest of this report, "streamflows" will refer to daily mean streamflow unless specified otherwise, and "historical streamflows" will refer to streamflow estimated for the period before installation of the index-velocity streamgages at Stillwater (water years 1910–2011) and Prescott (water years 1910–2007).

Physical Setting

The St. Croix Basin encompasses 7,730 square miles (mi^2) in Minnesota and Wisconsin (Lenz and others, 2001) (fig. 1). Before European settlement, the St. Croix Basin was dominated by forests, peatlands, and prairie grasslands (Niemela and Feist, 2000; Payne and others, 2002). Starting in the mid-1800s, logging, agricultural development, and urbanization altered the landscape, increased stormwater runoff, and created the need for more effective wastewater treatment in the basin. Today (2014), the St. Croix Basin is dominated by forest, pastures, and croplands, with most of the urban lands concentrated in the areas around the lowest 25 mi of the St. Croix River (Heiskary and Vavricka 1993, Larson and others, 2002). In addition, the St. Croix National Scenic Riverway provides 252 river miles for recreational use. Recreational use in the St. Croix National Scenic Riverway doubled from 1973 to 1995, reaching nearly one million visitors annually (Robertson and Lenz, 2002).

The lower St. Croix River comprises the 51.9 river miles from St. Croix Falls, Wisc. to the confluence with the Mississippi River at Prescott, Wisc. The final 25 mi of the St. Croix River, from Stillwater, Minn., to Prescott, Wisc., are referred to as Lake St. Croix because of near-zero slope, maximum water depths of more than 70 feet (ft), and a mean width greater than 0.3 mi (Robertson and Lenz, 2002). Several tributaries enter the lower St. Croix River, including the Apple

River upstream from Stillwater, and the Willow and Kinnickinnic Rivers downstream from Stillwater and upstream from Prescott (fig. 1).

Before installation of the index-velocity streamgages, streamflows for the inlet to and outlet from Lake St. Croix were estimated using streamflow-routing equations involving continuous streamflow records from upstream and tributary streamgages not affected by backwater conditions (LaFrancois and others, 2009; Magdalene and others, 2013). Streamflows of the St. Croix River at Stillwater were estimated using continuous streamflow data from an upstream streamgage at the St. Croix River at St. Croix Falls, Wisc. (USGS streamgage 05340500; hereafter referred to as St. Croix Falls) and a streamgage on the Apple River near Somerset, Wisc. (USGS streamgage 05341500; hereafter referred to as Apple River) (fig. 1; LaFrancois and others, 2009; Magdalene and others, 2013). Streamflows of the St. Croix River at Prescott were estimated using continuous streamflow data from the records at St. Croix Falls and two tributary streamgages: Willow River at Willow River State Park near Burkhardt, Wisc. (USGS streamgage 05341752; hereafter referred to as Willow River) and Kinnickinnic River near River Falls, Wisc. (USGS streamgage 05342000; hereafter referred to as Kinnickinnic River) (fig. 1, LaFrancois and others, 2009; Magdalene and others, 2013).

Methods

This section of the report describes various methods used in the development of regression equations to estimate historical streamflows for the St. Croix River at Stillwater and Prescott. Previous equations used for estimating historical streamflows are presented first, followed by methods for measurement of continuous streamflow at the index-velocity streamgages at Stillwater and Prescott. Methods for development of new regression equations and methods used to evaluate previous and new equations are then described. All equations used to estimate historical streamflows at Stillwater and Prescott are presented in table 1–1 in the appendix. All streamflow variables used in this report are in units of cubic feet per second.

Previous Equations for Estimating Historical Streamflows

Streamflow data from several USGS streamgages were used in previous equations for estimating historical streamflows of the St. Croix River at Stillwater and Prescott (table 1).

All USGS streamflow data are available from the National Water Information System database (NWIS; U.S. Geological Survey, 2014). Streamflow measurements by the USGS follow methods of Rantz and others (1982).

Historical streamflows at Stillwater ($Q_{Stillwater}$) were estimated in previous studies (Kloiber, 2004; Triplett and others, 2009; LaFrancois and others, 2009) using the equation:

$$Q_{Stillwater} = Q_{SCF} + Q_{Apple} \quad (1)$$

when streamflow data were available for both St. Croix Falls (Q_{SCF}) and Apple River (Q_{Apple}). However, the streamflow record at St. Croix Falls is more extensive than the streamflow

Table 1. Streamgaging sites and corresponding date ranges for data used in developing regression equations and comparing the abilities of previous equations and new regression equations to estimate measured streamflows at Stillwater, Minnesota (U.S. Geological Survey streamgage 05341550) and Prescott, Wisconsin (U.S. Geological Survey streamgage 05344490).

[Station number, the station number used to identify streamgages in the National Water Information System (*http://waterdata.usgs.gov/nwis/*); Station name, the descriptive name associated with the corresponding station number; Start date, the start date of data from the corresponding station used in presented analyses; End date, the end date of data from the corresponding station used in presented analyses]

Station number (fig. 1)	Station name	Start date	End date
05340500	St. Croix River at St. Croix Falls, Wisconsin	01/01/1910	09/30/2013
05341500	Apple River near Somerset, Wisconsin	09/30/2011	09/30/2013
05341550	St. Croix River at Stillwater, Minnesota	10/01/2011	09/30/2013
05341752	Willow River at Willow R State Park nr Burkhardt, Wisconsin	10/01/2012	09/30/2013
05342000	Kinnickinnic River near River Falls, Wisconsin	10/01/2012	09/30/2013
05344490	St. Croix River at Prescott, Wisconsin	10/01/2007	09/30/2013
05331000	Mississippi River at St. Paul, Minnesota	09/30/2007	09/30/2013
[1]NA	Discharge from Metropolitan Wastewater Treatment Facility	09/30/2007	09/30/2013
05344500	Mississippi River at Prescott, Wisconsin	10/01/2007	09/30/2013
05331580	Mississippi River Below L & D #2 at Hastings, Minnesota	10/01/2007	09/30/2013

[1]Site number assigned, data available as a data descriptor within 05331000 (internal only).

record for Apple River. Therefore, when Apple River data were unavailable, the mean ratio between Apple River streamflow and St. Croix Falls streamflow was used to estimate historical streamflows at Stillwater using only measured streamflow data from St. Croix Falls. On average, streamflows at Apple River were 11 percent of the streamflows at St. Croix Falls, resulting in the following equation (Kloiber, 2004; Triplett and others, 2009; LaFrancois and others, 2009):

$$Q_{Stillwater} = 1.11 * Q_{SCF} \quad (2)$$

For other USGS streamflow-routing equations used occasionally in project-specific situations, measured streamflows from St. Croix Falls and Apple River were lagged 1 day ($Q_{SCF\_lagged\_1\_day}$ and $Q_{Apple\_lagged\_1\_day}$, respectively) and summed (Greg Mitton, U.S. Geological Survey, written commun., 2014), resulting in the following equation:

$$Q_{Stillwater} = Q_{SCF\_lagged\_1\_day} + Q_{Apple\_lagged\_1\_day} \quad (3)$$

When streamflow data were available for Willow River (Q_{Willow}) and Kinnickinnic River ($Q_{Kinnickinnic}$), previous studies used estimated streamflows at Stillwater and measured streamflows at Willow River and Kinnickinnic River to estimate historical streamflows at Prescott ($Q_{Prescott}$) using equation 4 (Kloiber, 2004; Triplett and others, 2009; LaFrancois and others, 2009).

$$Q_{Prescott} = Q_{Stillwater} + Q_{Willow} + Q_{Kinnickinnic} \quad (4)$$

However, the overlapping period of record for Willow River and Kinnickinnic River was brief, so the mean streamflow ratios of Willow River to St. Croix Falls (0.037) and Kinnickinnic River to St. Croix Falls (0.032) were added to the estimated multiplier of equation 2 for Stillwater to produce the following equation for estimating historical streamflows at Prescott (Kloiber, 2004; Triplett and others, 2009; LaFrancois and others, 2009):

$$Q_{Prescott} = 1.18 * Q_{SCF} \quad (5)$$

During certain past flood situations, USGS hydrographers calculated the Prescott streamflow using data from USGS streamgages on the Mississippi River and measured discharges from the Metropolitan Wastewater Treatment Facility (Q_{MWTF}), which were collected by MCES (Jinger Pulkrabek, MCES, written commun., 2014; data not publicly available). Streamflow at Prescott was determined by summing the streamflow from the Mississippi River at St. Paul, Minn. (USGS streamgage 05331000; $Q_{St.Paul}$) and Q_{MWTF}, lagging that sum by 1 day [$(Q_{St.Paul} + Q_{MWTF})_{lagged\_1\_day}$], and multiplying by a constant to account for unmeasured portions of the flow in the Mississippi River. The resulting total then was subtracted from the streamflow of the Mississippi River at Prescott, Wisc. (USGS streamgage 05344500; $Q_{Miss\_Prescott}$) (Greg Mitton, U.S. Geological Survey, written commun., 2014), resulting in the following equation to calculate streamflow of the St. Croix River at Prescott ($Q_{Prescott}$):

$$Q_{Prescott} = Q_{Miss\_Prescott} - \{1.034[(Q_{St.Paul} + Q_{MWTF})_{lagged\_1\_day}]\} \quad (6)$$

Measurement of Continuous Streamflow at Index-Velocity Streamgages

The index-velocity streamgages at Prescott (USGS streamgage 05344490) and Stillwater (USGS streamgage 05341550) have collected continuous streamflow data since 2007 and 2011, respectively (U.S. Geological Survey, 2014). The index-velocity streamgage at Prescott (USGS streamgage 05344490) was installed by USGS personnel on August 20, 2007. At Stillwater, an existing U.S. Army Corps of Engineers (USACE) stage-only streamgage was converted to an index-velocity streamgage on September 9, 2011, when USGS personnel added an acoustic Doppler velocity meter (ADVM) to collect continuous velocity data. Continuous stage and velocity data were collected at Prescott and Stillwater using methods and instrumentation in accordance with USGS protocols (Rantz and others, 1982; Kennedy, 1983; Fallon and others, 2002; Mueller and others, 2013; Turnipseed and Sauer, 2010). Continuous streamflow data were calculated using stage-discharge and index-velocity rating curves developed according to methods specified in Levesque and Oberg (2012). Continuous streamflow records were worked, checked, and reviewed according to protocols described in Fallon and others (2002) and Levesque and Oberg (2012). Interruptions in continuous data collection because of ice cover or equipment malfunction caused a few gaps in the continuous streamflow record, and daily streamflows during these gaps were estimated by USGS hydrographers (U.S. Geological Survey, 2014). Estimated streamflow values were not used for the regression analyses presented in this report. Daily streamflow data (U.S. Geological Survey, 2014) from water years 2012 and 2013 for the Stillwater index-velocity streamgage and from water years 2008 through 2013 for the Prescott index-velocity streamgage were used in this study.

Methods for Development of New Equations to Estimate Streamflows

Simple and multiple linear regression analyses were used to evaluate the streamflow relations between the upstream and tributary streamgages and the new index-velocity streamgages at Stillwater and Prescott. Several regression equations with various explanatory variables were compared for Stillwater and Prescott to account for differing periods of operation for upstream and tributary streamgages (table 1). However, St. Croix Falls provides the longest and most consistent streamflow record for predicting downstream streamflows at Stillwater and Prescott. Model goodness-of-fit was evaluated using the Nash-Sutcliffe Efficiency (NSE; Nash and Sutcliffe, 1970) combined with graphical analyses as described in the following section. The best regression equations for estimating streamflows at Stillwater and at Prescott were selected based on NSE values and were used to calculate revised estimates of historical streamflows at Stillwater and Prescott.

Methods for Evaluation of Equations Used to Estimate Historical Streamflows

The goodness-of-fit of previous and new equations for estimating daily streamflows at Stillwater and Prescott were compared using the NSE (Nash and Sutcliffe, 1970), which is defined in equation 7;

$$NSE = 1 - \frac{\sum_{i=1}^{n} (Q_{m,i} - Q_{e,i})^2}{\sum_{i=1}^{n} (Q_{m,i} - \bar{Q}_m)^2} \quad (7)$$

where $Q_{m,i}$ is the measured daily mean streamflow at the index-velocity streamgage for each day i, n is the upper bound of the summation, $Q_{e,i}$ is the estimated daily mean streamflow from the model for each day i, and is the mean of all measured daily streamflows for the examined period of record. The NSE is a normalized statistic that determines the relative magnitude of the residual variance compared to measured data variance (Nash and Sutcliffe, 1970). Furthermore, the NSE indicates how well the plot of the relation between observed (measured) and simulated (modeled) data fits a 1:1 line. When NSE is equal to 1, the simulated data match the observed data perfectly. When NSE is equal to 0, the simulated data are as accurate as the mean of the observed data. Finally, when NSE is less than 0, the observed mean is a better estimator than the model. Unlike the NSE, the more commonly used coefficient of determination (R^2) does not expect a 1:1 line fit. However, for the model development analyses presented in this report, the NSE values are equivalent to calculated R^2 values.

Methods for Calculation of Revised Estimates of Historical Streamflows

New regression equations developed in this study were used to calculate revised estimates of historical streamflows of the St. Croix River at Stillwater and Prescott from measured streamflows at St. Croix Falls. Revised historical estimates were compared to previous historical estimates derived from equations 2 and 5 for Stillwater and Prescott, respectively (Kloiber, 2004; Triplett and others, 2009; LaFrancois and others, 2009). Estimates of total monthly streamflow were compared using previous and new regression equations to facilitate future comparisons of estimated historical nutrient loads at Stillwater and Prescott.

Methods for Evaluation of Revised Estimates of Historical Streamflows

The accuracies of new regression equations for estimating historical streamflows of the St. Croix River at Stillwater and Prescott were evaluated by comparing historical estimates of streamflow to discrete streamflow measurements made before the installation of index-velocity streamgages. Estimated daily streamflows and discrete measured streamflows were compared using the percent difference equation:

$$Percent\ Difference = \left[\frac{Q_e - Q_m}{Q_e}\right] * 100 \quad (8)$$

where Q_e is the estimated daily mean streamflow and Q_m is the discrete measured streamflow. Equation 8 was chosen to calculate percent differences to obtain negative values when estimated streamflows were less than measured streamflows and to obtain positive values when estimated streamflows were greater than measured streamflows. At Stillwater, nine discrete streamflow measurements were made from 2001 through 2011, and stage data from the USACE stage-only streamgage were available for all measurements. Therefore, the ranges of stages observed on the days of the discrete measurements were used to assess how variability in daily streamflow may affect calculated differences between discrete streamflow measurements and daily estimates of streamflow. However, stage data were not available for discrete streamflow measurements at Prescott, so the potential effects of daily variability in streamflow on the calculated differences between discrete measurements and daily estimates of streamflow could not be examined for that location. At Prescott, 16 discrete streamflow measurements were made between 1928 and 2007. However, no measurements were made between 1972 and 2000. Little information exists about the methods used to make discrete measurements in 1972 and earlier, but the methods used to measure discharge may not have been as accurate as current hydroacoustic methods (Rantz and others, 1982; Levesque and Oberg, 2012).

After calculating previous and revised estimates of historical daily streamflows for the St. Croix River at Stillwater and Prescott from 1910 to the start of index-velocity records (appendixes 2 and 3, respectively), general comparisons were made between previous and revised estimates. Comparisons included streamflows during high-flow months (March through June) and low-flow months (July through September), streamflows during high-flow and low-flow years, streamflows throughout 30-year dry and wet periods, and mean streamflows throughout the entire period of estimated record. Because of serial correlation, additional significance tests, confidence intervals, and estimates of variance were not determined.

Development of Regression Equations to Estimate Historical Streamflows

New regression equations were developed to improve historical streamflow estimates of the St. Croix River at Stillwater and Prescott. The new regression equations are presented in this section of the report along with an evaluation of their accuracy compared to previous equations.

Regression Equations for St. Croix River at Stillwater, Minnesota

Published continuous streamflow records (U.S. Geological Survey, 2014) for St. Croix Falls (USGS streamgage 05340500), Apple River (USGS streamgage 05341500), and the recently (2011) installed Stillwater index-velocity streamgage (USGS streamgage 05341550) were used to compare previous equations for estimating historical streamflow and develop a new linear regression equation that improves estimates of historical streamflows of the St. Croix River at Stillwater. Several candidate regression equations were developed using combinations of the following explanatory variables: Q_{SCF}, $Q_{SCF\_lagged\_1\_day}$, Q_{Apple}, and $Q_{Apple\_lagged\_1\_day}$. The simplest and most accurate regression equation (based on NSE values) for estimating streamflow at Stillwater is shown in equation 9.

$$Q_{Stillwater} = (0.968 * Q_{SCF\_lagged\_1\_day}) + 770 \qquad (9)$$

Equation 9 can be used to compute revised estimates of historical streamflows at Stillwater.

The abilities of previous regression equations (equations 1–3) and the new regression equation (equation 9) to estimate streamflows at Stillwater were compared using the measured streamflow record for the index-velocity streamgage at Stillwater (water years 2012 and 2013) and NSE values. The NSE values for selected regression equations are presented in table 2.

Table 2. Nash-Sutcliffe Efficiencies that evaluate the abilities of selected regression equations to fit measured streamflows (water years 2012–13) of the St. Croix River at Stillwater, Minnesota (U.S. Geological Survey streamgage 05341550).

[Equation number, the number of the corresponding equation in the body of the report and listed in table 1–1; Figure label, the label corresponding to a time-series plot in figure 2; *A*–*D*, letters of corresponding time-series plots in figure 2; Number of values used, number of data points used to calculate NSE values; NSE, Nash-Sutcliffe Efficiency (Nash and Sutcliffe, 1970)]

Equation number	Figure label	Number of values used[1]	NSE
1	*A*	444	0.913
2	*B*	444	0.896
3	*C*	443	0.970
9	*D*	444	0.979

[1]Estimated values in published streamflow records (U.S. Geological Survey, 2014) were omitted from analyses.

All compared equations did a reasonably good job of accurately fitting measured streamflow at Stillwater, with NSE values of 0.896 or greater (table 2). Incorporating a 1-day time lag (equations 3 and 9) resulted in the largest increase in NSE values among equations. However, the developed regression equation (equation 9) estimated streamflow at Stillwater better than previous equations (equations 1–3), which generally overestimated peak flows and underestimated base flows (fig. 2). In addition, because the period of continuous record for the St. Croix Falls streamgage (05340500) extends back more to 1910, equation 9 likely produces the most accurate estimates for the longest period of time.

The constant term and 1-day time lag in equation 9 may produce more accurate streamflow estimates for several reasons. The constant term may account for a certain amount of base flow or groundwater contributions between St. Croix Falls and Stillwater. During low flows, the actual time lag may be longer than 1 day, but wind and wave action may cause enough variation in flows to mix out the effects of the time lag; furthermore, the constant term may help dampen the variations caused by wind and wave action at lower flows. The improvements in estimation of high flows with the time lag indicates that the time lag may be especially important at high flows because of the rapid rate of change in flows and water levels. Any effects of increased flow at Apple River are negated by the more dramatic increases in flow at St. Croix Falls. Finally, water velocities and time of travel likely are strongly affected by hydrologic damming at Prescott.

Because only 2 full years of measured streamflow data from the index-velocity streamgage were available for analysis of the goodness-of-fit for the regression equations developed for Stillwater, additional analyses were used to evaluate the regression method for a variety of flow regimes. Separate regression equations were developed for water years 2012 and 2013 using streamflow at St. Croix Falls with a 1-day time lag ($Q_{SCF\_lagged\_1\_day}$) as the explanatory variable and streamflow at Stillwater as the response variable. Equation 10 was developed using streamflow data from water year 2013 data and was used to estimate streamflows at Stillwater for water year 2012 [$Q_{Stillwater(WY2012)}$].

$$Q_{Stillwater(WY2012)} = (0.960 * Q_{SCF\_lagged\_1\_day}) + 907 \qquad (10)$$

Equation 11 was developed using streamflow data from water year 2012 and was used to estimate streamflows at Stillwater for water year 2013[$Q_{Stillwater(WY2013)}$].

$$Q_{Stillwater(WY2013)} = (0.971 * Q_{SCF\_lagged\_1\_day}) + 680 \qquad (11)$$

The NSE values used to evaluate the abilities of equations 10 and 11 to estimate streamflow for the other water year are presented in table 3, and time-series plots comparing measured and estimated streamflows are presented in figure 3. The NSE

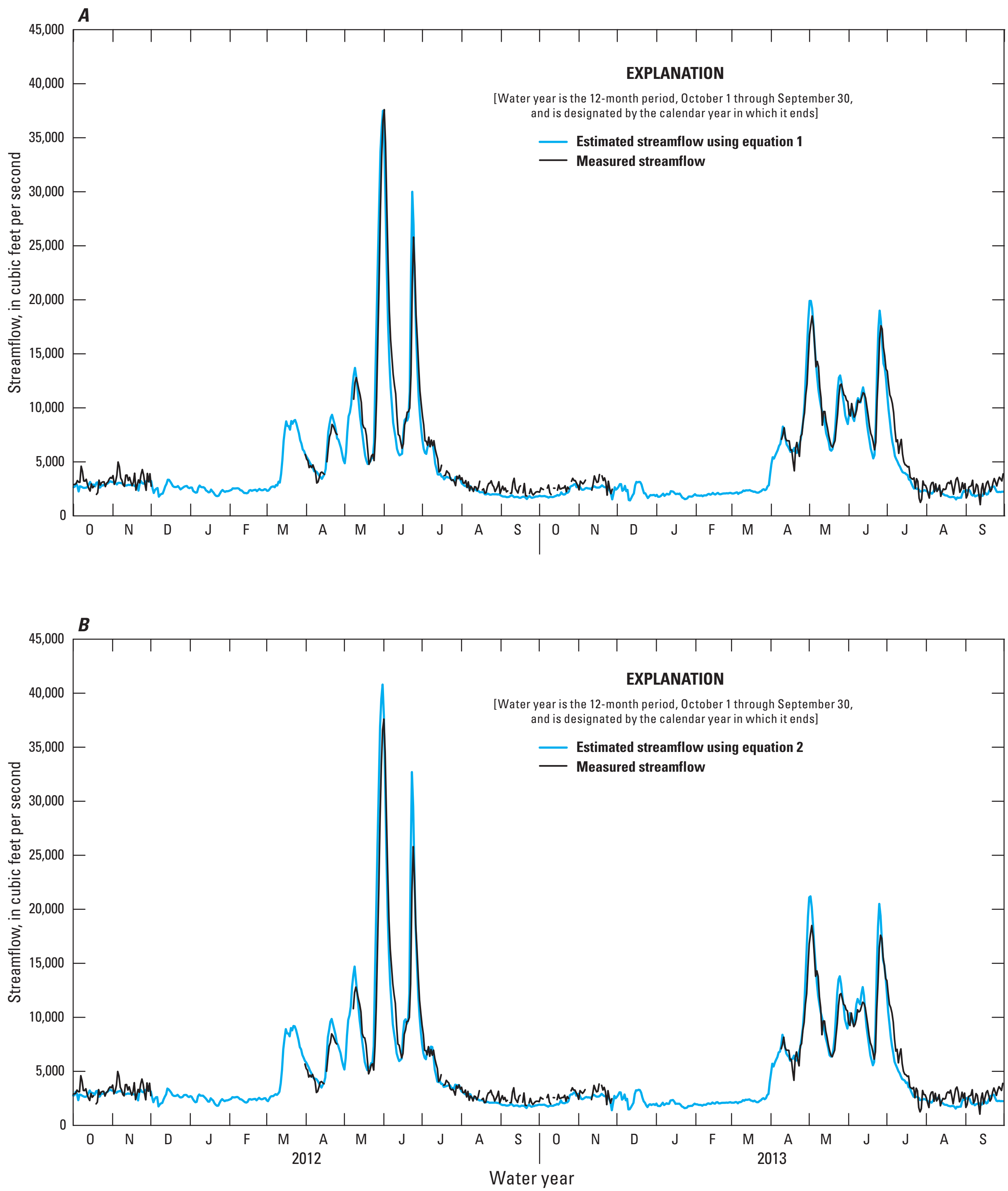


Figure 2. Time-series plots used to visually compare measured streamflows (water years 2012–13) for the St. Croix River at Stillwater, Minnesota (U.S. Geological Survey streamgage 05341550) to streamflows estimated using selected regression equations from table 1. *A*, equation 1; *B*, equation 2; *C*, equation 3; and *D*, equation 9.

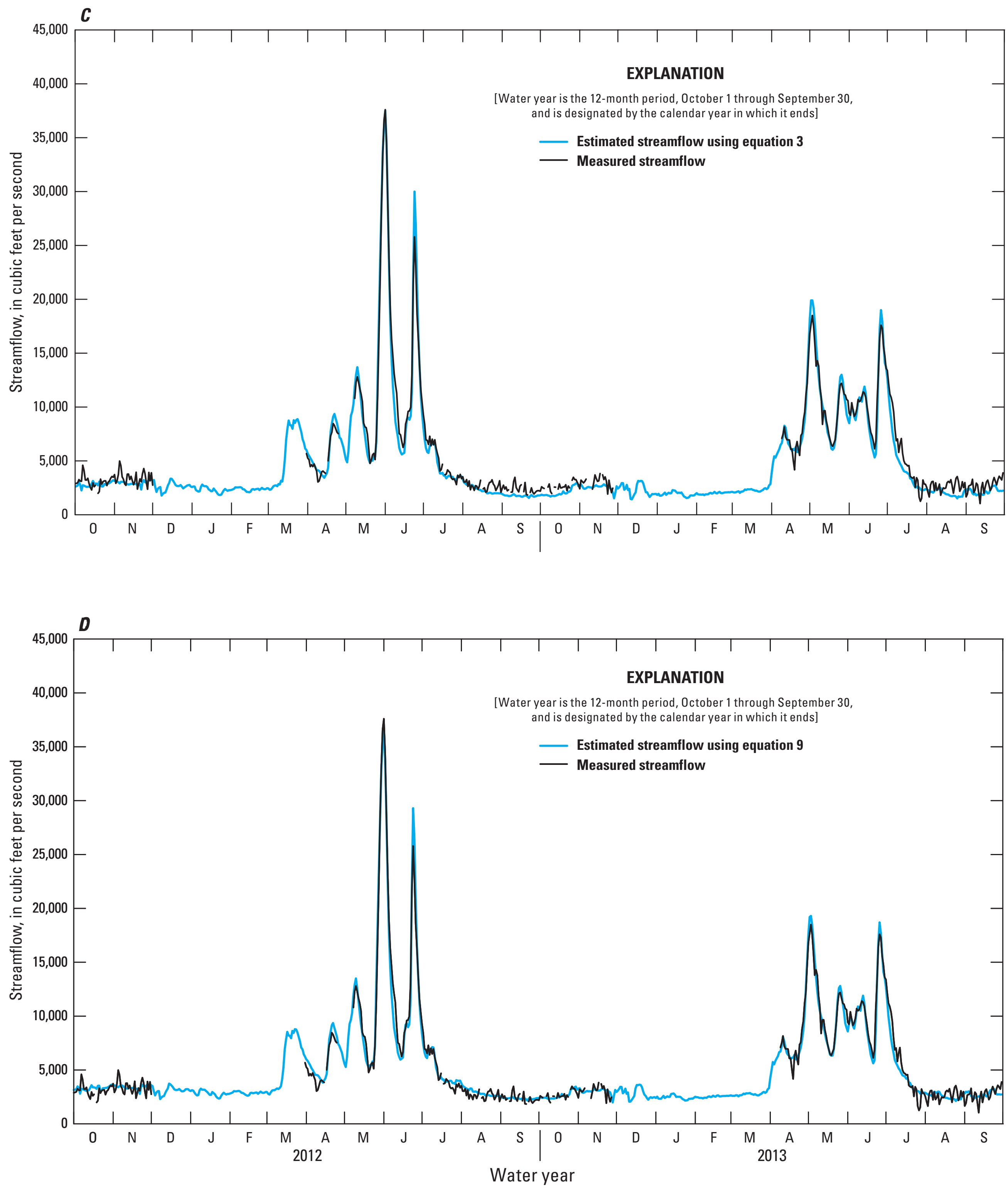


Figure 2. Time-series plots used to visually compare measured streamflows (water years 2012–13) for the St. Croix River at Stillwater, Minnesota (U.S. Geological Survey streamgage 05341550) to streamflows estimated using selected regression equations from table 1. *A*, equation 1; *B*, equation 2; *C*, equation 3; and *D*, equation 9.—Continued

Table 3. Nash-Sutcliffe Efficiencies for regression equations developed using measured streamflows of the St. Croix River at St. Croix Falls (U.S. Geological Survey streamgage 05340500) from one water year to fit measured streamflows for another water year at the St. Croix River at Stillwater, Minnesota (U.S. Geological Survey streamgage 05341550) to validate the regression method used to develop equation 9 for estimating historical streamflows at the St. Croix River at Stillwater, Minnesota.

[Equation number, the number of the corresponding equation in the body of the report and listed in table 1–1; Figure label, the label corresponding to a time-series plot in figure 3; *A–B*, letters of corresponding time-series plots in figure 3; Development year, the water year of the measured streamflow data used to develop the corresponding regression equation; Prediction year, the water year of measured streamflow data being estimated using the regression equation derived from the development year; Number of values used, the number of daily streamflow values in the prediction year that were used in calculating NSE values; NSE, Nash-Sutcliffe Efficiency (Nash and Sutcliffe, 1970)]

Equation number	Figure label	Development year	Prediction year	Number of values used[1]	NSE
10	*A*	2013	2012	239	0.981
11	*B*	2012	2013	223	0.970

[1]Estimated values in published streamflow records (U.S. Geological Survey, 2014) were omitted from analyses.

values and the time-series plots demonstrate that the regression method is fairly robust throughout varied streamflow conditions.

Regression Equations for St. Croix River at Prescott, Wisconsin

Published continuous streamflow records (U.S. Geological Survey, 2014) for St. Croix Falls (USGS streamgage 05340500), Stillwater (USGS streamgage 05341550), Willow River (USGS streamgage 05341752), Kinnickinnic River (USGS streamgage 05342000), and the recently (2007) installed Prescott index-velocity streamgage (USGS streamgage 05344490) were used to compare previous equations for estimating historical streamflows and develop a new linear regression equation that can be used to improve historical streamflow estimates for the St. Croix River at Prescott. The following explanatory variables were used to construct several candidate linear regression equations: Q_{SCF}, Q_{Apple}, Q_{Willow}, $Q_{Kinnickinnic}$, $Q_{Miss\_Prescott}$, $Q_{St.Paul}$, Q_{MWTF}. Candidate regression equations incorporated various combinations of time lags for included explanatory variables, and incorporated time lags ranged from 1 to 4 days. The simplest and most accurate regression equation (based on NSE values) for estimating streamflow at Prescott used and as the two explanatory variables:

$$Q_{Prescott} = (0.585 * Q_{SCF\_lagged\_1\_day}) + (0.415 * Q_{SCF\_lagged\_3\_day}) + 873 \quad (12)$$

Equation 12 can be used to compute revised estimates of historical streamflows for the St. Croix River at Prescott. The two separate time lags may account for responses to precipitation events in different parts of the basin and changes in water storage between Stillwater and Prescott. The abilities of previous and revised equations to estimate streamflow at Prescott were compared using measured streamflow record at Prescott (water years 2008 through 2013) and NSE values. The NSE values for selected predictor models are presented in table 4.

The NSE values from equations used to estimate streamflows at Prescott ranged from 0.708 to 0.941 (table 4), indicating that streamflow at Prescott is more difficult to accurately estimate than streamflow at Stillwater (table 2). In addition to the dynamics of water storage upstream in Lake St. Croix, streamflow of the St. Croix River at Prescott is affected immediately downstream by the confluence with the Mississippi River. The hydrology of the Mississippi River is complex because the channel is maintained for commercial navigation and controlled by many dams that hold or release water for a variety of uses (Upper Mississippi River Basin Commission, 1982; Lubinski and others, 1991), including Lock and Dam No. 2 (not shown) above Prescott and Lock and Dam No. 3 (not shown) below Prescott. In addition, the Mississippi River is affected upstream from Prescott by the confluence with the Minnesota River (fig. 1). Most of the Minnesota River Basin occupies lower latitudes than the Upper Mississippi and St. Croix River Basins (Upper Mississippi River Basin Commission, 1982; Lubinski and others, 1991). Therefore, precipitation events in the Minnesota River Basin can strongly affect the hydrologic damming that occurs at Prescott even when precipitation events do not occur in the Upper Mississippi and St. Croix River Basins. Furthermore, Mississippi River streamflows measured downstream from the confluence with the Minnesota River (USGS streamgage 05331000) can be double the streamflows measured upstream from the confluence with the Minnesota River (USGS streamgage 05288500; U.S. Geological Survey, 2014).

The newly developed regression equation (equation 12) estimated streamflow at Prescott better than previous equations (equations 4–6) based on NSE values (table 4) and graphical plots (fig. 4). Equation 6 does a reasonable job

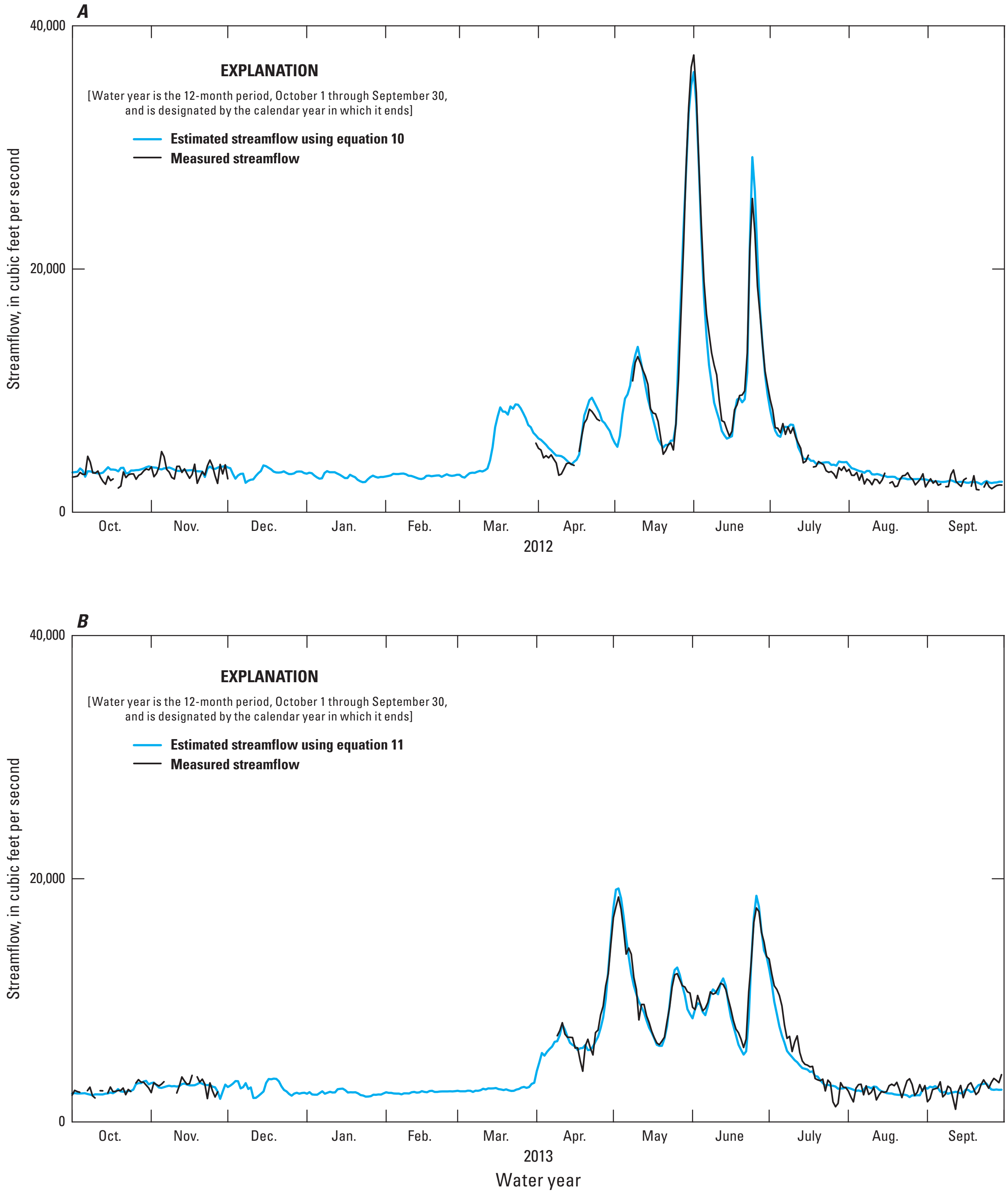


Figure 3. Time-series plots of measured and estimated streamflow for the St. Croix River at Stillwater, Minnesota, using regression equations based on streamflow data from the St. Croix River at St. Croix Falls (U.S. Geological Survey streamgage 05340500) for 2 water years. *A*, estimated streamflow for water year 2012 based on records from water year 2013 (equation 10, table 1–1); and *B*, estimated streamflow for water year 2013 based on records from water year 2012 (equation 11, table 1–1).

Table 4. Nash-Sutcliffe Efficiencies that evaluate the abilities of selected regression equations equations to fit measured streamflows (water years 2008–13) of the St. Croix River at Prescott, Wisconsin (U.S. Geological Survey streamgage 05344490 for water years 2008–13 records).

[Equation number, the number of the corresponding equation in the body of the report and listed in table 1–1; Figure label, the label corresponding to a time-series plot in figure 4; *A–D*, letters of corresponding time-series plots in figure 4; Number of values used, the number of daily streamflow values used in calculating NSE values; NSE, Nash-Sutcliffe Efficiency (Nash and Sutcliffe, 1970)]

Equation number	Figure label	Number of values used[1]	NSE
4	*A*	[2]212	0.758
5	*B*	1,733	0.774
6	*C*	1,255	0.708
12	*D*	1,752	0.941

[1]Estimated values in published streamflow records (U.S. Geological Survey, 2014) were omitted from analyses.

[2]Only streamflow data for water year 2013 were available for the Willow River at Willow River State Park near Burkhardt, Wisconsin (U.S. Geological Survey streamgage 05341752).

estimating peak discharges. However, at low flows, streamflows estimated using equation 6 often were substantially overestimated or actually negative (fig. 4*C*). In contrast, equation 5 consistently underestimates base flow and substantially overestimates peak flows (fig. 4*B*). Because the streamflow record for Willow River is brief (only water year 2013) and the Stillwater component for equation 4 is estimated before September 2011, equation 4 is the least useful predictor equation. Although a measured streamflow record at Stillwater is available for water year 2013, equation 2 was used to estimate the streamflow at Stillwater used in equation 4 (table 4; fig. 4) to match previously used historical estimates of streamflow. When the measured streamflow record from Stillwater was used in equation 4, the NSE value increased from 0.758 (table 4) to 0.897. However, even when the measured Stillwater streamflows are used, equation 4 generally underestimates low flows. Equation 12 most accurately estimates measured discharges at Prescott and does not consistently overestimate peak flows and underestimate low flows (fig. 4*D*). In addition, because the St. Croix Falls record extends back more than 100 years, equation 12 likely produces the most accurate estimates for the longest period of time.

Computation and Evaluation of Revised Estimates of Historical Streamflows

Revised estimates of historical streamflows for the St. Croix River at Stillwater and Prescott were computed and are described in this section of the report. The revised estimates of historical streamflows were evaluated using discrete streamflow measurements.

Computation of Revised Historical Streamflows for the St. Croix River at Stillwater, Minnesota

The newly developed regression equation (equation 9) was used to compute revised estimates of historical streamflows for the St. Croix River at Stillwater for water years 1910–2011. The revised estimates of historical streamflows for Stillwater were compared to previous estimates of streamflows that were calculated using equation 2. Although equation 2 was not the most accurate equation of the previous estimation equations (table 2), equation 2 provided a consistent estimation method for the longest period of record and was the equation most used in previous studies (Kloiber, 2004; Triplett and others, 2009; LaFrancois and others, 2009). Annual comparisons of previous and revised estimates of historical streamflows at Stillwater are presented in appendix 2.

Because of the short duration of the continuously measured streamflows at the index-velocity streamgage at Stillwater (water years 2012–13), some measured streamflows at St. Croix Falls (explanatory variable) used in the computation of revised historical streamflows at Stillwater (water years 1910–2011) were outside the range of streamflows used to develop equation 9, which subsequently was used to generate revised estimates of historical streamflows at Stillwater. However, the new regression equation still incorporates 95 percent of the range of daily streamflows measured at St. Croix Falls for the historical streamflow period (water years 1910–2011) (table 5). Although the accuracy of the equation 9 cannot be evaluated outside of the range of measured streamflow at St. Croix Falls used in the analysis, equation 9 likely provides an improvement compared to previous methods of estimating historical streamflows (equations 1–3). Selected statistics used to evaluate the regression datasets in relation to the period of record at St. Croix Falls for Stillwater are listed in table 5.

Computation of Revised Historical Streamflows for the St. Croix River at Prescott, Wisconsin

The newly developed regression equation (equation 12) was used to compute revised estimates of historical streamflows for the St. Croix River at Prescott for water years 1910–2007. Revised estimates of historical streamflows for Prescott were compared to previous estimates of historical streamflows that were calculated using equation 5 for several reasons. First, equation 5 was the most accurate of the previous regression equations (table 4). Second, equation 5 provided a consistent estimation method for the longest period of record. Third, equation 5 was used the most in previous studies (Kloiber, 2004; Triplett and others, 2009; LaFrancois and others, 2009). Annual comparisons of previous and revised

Table 5. Selected statistics for measured streamflows of the St. Croix River at St. Croix Falls, Wisconsin (U.S. Geological Survey streamgage 05340500) used to compute revised estimates of historical streamflows for the St. Croix River at Stillwater, Minnesota and Prescott, Wisconsin, and minimum and maximum estimated streamflows at Stillwater and Prescott using new regression equations.

[Site of estimated streamflow, location for which new regression equations were applied to improve estimates of historical streamflows; ft^3/s, cubic feet per second]

Site of estimated streamflow	Minimum streamflow for explanatory variable[1] (ft^3/s)	Percentage historical streamflow[2] below minimum streamflow for explanatory variable[1]	Maximum streamflow for explanatory variable[1] (ft^3/s)	Percent of historical streamflow[2] above maximum streamflow for explanatory variable[1]	Percent of historical streamflow[2] within range of streamflows for explanatory variable[1]	Minimum estimated streamflow (ft^3/s)	Maximum estimated streamflow (ft^3/s)
St. Croix River at Stillwater, Minnesota	[3]1,400	[3]5.25	[3]36,800	[3]0.14	[3]94.6	[5]2,120	[5]36,400
St. Croix River at Prescott, Wisconsin	[4]1,130	[4]1.44	[4]36,800	[4]0.15	[4]98.4	[6]2,020	[6]36,100

[1]Explanatory variable is QSCF_lagged_1_day, which represents streamflow of the St. Croix River at St. Croix Falls, Wisconsin (U.S. Geological Survey streamgage 05340500) lagged by 1 day.

[2]Historical streamflow is measured streamflow of the St. Croix River at St. Croix Falls, Wisconsin (U.S. Geological Survey streamgage 05340500) for water years 1910–2011 for St. Croix River at Stillwater, Minnesota, and for water years 1910–2007 for St. Croix River at Prescott, Wisconsin.

[3]Period of record for explanatory variable used in developing regression equation for St. Croix River at Stillwater, Minnesota, was water years 2012–13.

[4]Period of record used for explanatory variable in developing regression equation for St. Croix River at Prescott, Wisconsin, was water years 2008–13.

[5]Period of estimated streamflow is water years 1910–2011.

[6]Period of estimated streamflow is water years 1910–2007.

estimates of historical streamflows at Prescott are presented in appendix 3.

Because of the relatively short duration of continuously measured streamflows at the index-velocity streamgage at Prescott (water years 2008-2013), some measured streamflows at St. Croix Falls (explanatory variable) used in the computation of revised historical streamflows at Prescott (water years 1910–2007) were outside the range of streamflows used to develop equation 12. However, the new regression equation still incorporates 98 percent of daily streamflows measured at St. Croix Falls for the historical streamflow period (water years 1910–2007) (table 5). Although the accuracy of equation 12 cannot be evaluated outside the range of measured streamflows at St. Croix Falls used in the analysis, equation 12 likely improves estimates of extreme historical streamflows (equations 4–6). Selected statistics of data used to evaluate the regression datasets in relation to the period of record at St. Croix Falls for Prescott are listed in table 5.

Evaluation of Revised Estimates of Historical Streamflows at Stillwater, Minnesota

Revised estimates of historical discharges at Stillwater computed using equation 9 were compared to nine discrete streamflow measurements made from 2001 to 2011. Five of the measurements were made by USGS personnel, and the data are publicly available in the NWIS database (U.S. Geological Survey, 2014). The other four measurements were made by MCES staff (Scott Schellhaas, Metropolitan Council Environmental Services, written commun., 2014). Discrete streamflow measurements generally were made in the span of about an hour. The results of these comparisons are presented in table 6.

Percent-difference data in table 6 indicate that equation 9 does a fairly good job of estimating historical streamflows at Stillwater. Revised estimates of historical streamflows were within 10 percent of discrete streamflows measured by USGS hydrographers (appendix 2). In addition, revised estimates were within 18 percent of all discrete streamflow measurements.

Stage data (table 6) provide additional insight into the ability of equation 9 to estimate daily streamflows at Stillwater. During periods of falling stage, a discrete streamflow measured early in the day, intuitively, would be higher than the estimated daily streamflow. Six of the seven measurements made during falling stages had discrete streamflow measurements that were greater than daily estimated streamflows (table 6). Similarly, during periods of rising stage, a discrete streamflow measured early in the day, intuitively, would be lower than an estimated daily streamflow. For the single measurement made during a rising stage, the discrete streamflow measurement discharge was less than the estimated daily streamflow (table 6). For the one measurement made during a period of stable stage, the discrete streamflow measurement was within 2.1 percent of the estimated daily streamflow

Table 6. Comparisons between estimated historical daily mean streamflows and discrete streamflow measurements made before the installation of an index-velocity streamgage at the St. Croix River at Stillwater, Minnesota.

[Agency, the agency that made the discrete streamflow measurement with the corresponding date (USGS, U.S. Geological Survey; MCES, Metropolitan Council Environmental Services); measurement location (A, streamflow measurement location in a confined portion of the St. Croix River located 3 miles upstream from the Stillwater Lift Bridge; B, a streamflow measurement location 50 feet upstream from the Stillwater Lift Bridge); discrete *Qm*, discrete measured streamflow; ft^3/s, cubic feet per second; daily *Qe*, daily mean streamflow estimated using equation 9 in the text and listed in table 1–1 for the corresponding date; percent difference, the percent difference between the discrete *Qm* and the daily *Qe* estimated using equation 8 from the text and listed in table 1–1; range, the range of measured stage (minimum to maximum) for the corresponding date; stage pattern, the general pattern of the stage data for the corresponding date]

Date of discrete streamflow measurement	Time of discrete streamflow measurement	Agency	Measurement location	Discrete *Qm* (ft^3/s)	Daily *Qe* (ft^3/s)	Percent difference	Stage range (feet)	Stage pattern
09/18/2001	11:27	USGS	A	2,890	3,090	+6.47	0.08	Rising
10/04/2007	11:00	MCES	B	4,940	4,220	-17.1	0.18	Falling
10/30/2007	11:00	MCES	B	9,000	7,880	-14.2	0.41	Falling
06/17/2008	10:45	MCES	B	14,100	14,400	+2.08	0.03	Stable
04/21/2011	10:36	USGS	A	10,800	10,600	-1.89	0.33	Falling
05/06/2011	10:17	USGS	A	11,700	12,500	+6.40	0.16	Falling
06/06/2011	11:30	MCES	B	9,370	8,960	-4.58	0.28	Falling
07/28/2011	8:40	USGS	A	4,620	4,320	-6.94	0.22	Falling
07/28/2011	9:37	USGS	B	4,740	4,320	-9.72	0.22	Falling

(table 6). The percent-difference comparisons between daily estimated streamflows and discrete streamflow measurements provides additional evidence that the newly developed equation 9 can be used to accurately back-calculate historical streamflows at Stillwater.

The comparisons in table 6 indicate that equation 9 provides reasonable estimates of historical streamflows. However, the ability of equation 9 to accurately estimate historical streamflow before 2001 could not be evaluated because all previous discrete streamflow measurements were made within a relatively short (10-year) period of record. Conversely, the channel between St. Croix Falls and Stillwater is completely riverine, so factors like water storage within Lake St. Croix are less likely to confound the relation between St. Croix Falls and Stillwater throughout the period of record. Therefore, historical estimates of streamflows computed from equation 9 likely are reasonably accurate throughout the period of record for the St. Croix Falls streamgage (05340500) used to estimate streamflows at Stillwater (water years 1910–2011).

Comparisons of previous and revised estimates of historical streamflows at Stillwater using equations 2 and 9, respectively (appendix 2), indicate noticeable differences between streamflow estimation equations. Monthly and annual comparisons support the findings of the regression analyses that previous regression equations generally underestimated low flows and overestimated high flows compared to the new regression equation (fig. 2). During the relatively dry, low-flow period of water years 1910–40, the previously estimated mean streamflow of 3,600 cubic feet per second (ft^3/s) was 310 ft^3/s less than the revised estimated mean streamflow of 3,910 ft^3/s. Also, during the relatively wet, high-flow period of water years 1970–2000, the previously estimated mean streamflow of 5,560 ft^3/s was 60 ft^3/s less than the revised estimated mean streamflow of 5,620 ft^3/s. For the period of estimated historical streamflows (water years 1910–2011), the previously estimated mean streamflow of 4,850 ft^3/s was 150 ft^3/s less than the revised mean streamflow of 5,000 ft^3/s, indicating that over the historical period, overestimates of peak flows were not high enough to balance the underestimates of low-flow conditions at Stillwater.

The differences between previous and revised estimates of historical streamflows may demonstrate the effects of groundwater contributions on streamflows at Stillwater. Equation 2 used a simple scalar and generally underestimated low flows and overestimated peak flows in comparison to equation 9. The constant in equation 9 (y-intercept; 770) may represent a minimum base-flow value that was not accounted for in equation 2. In addition, the constant may better account for geologic differences between St. Croix Falls and Stillwater. Downstream from St. Croix Falls, increased soil and bedrock permeability may allow the surrounding watershed to capture a greater portion of the water budget (Juckem, 2007), resulting in less peak runoff and greater base flow. However, the hydrograph separation analyses needed to quantify base flow and peak flows at St. Croix Falls and Stillwater are beyond the scope of this report.

Evaluation of Revised Estimates of Historical Streamflows at Prescott, Wisconsin

Revised estimates of historical streamflows for the St. Croix River at Prescott computed using equation 12 were compared to 16 discrete streamflow measurements made from 1928 to 2007. Because daily streamflow estimates were

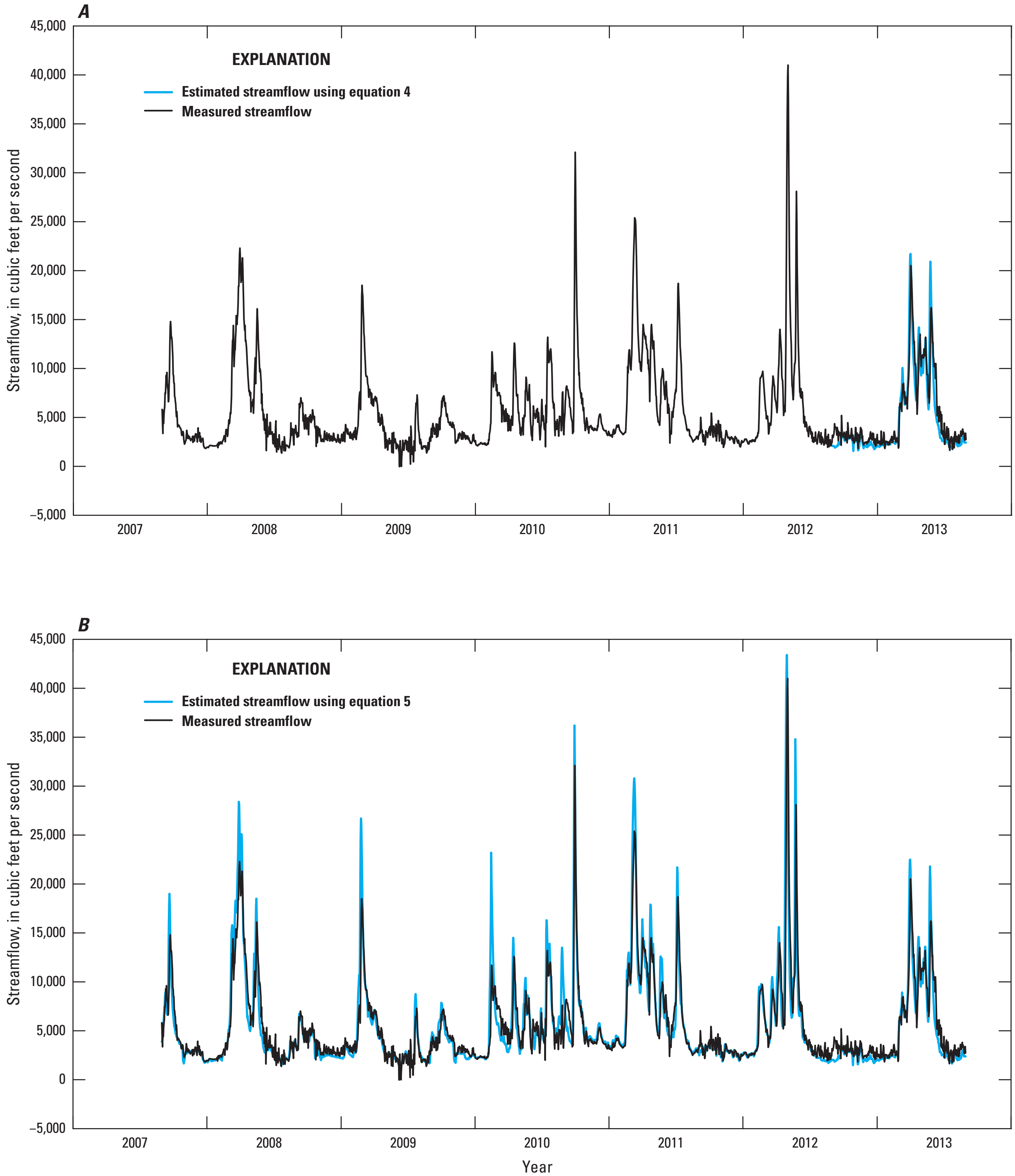
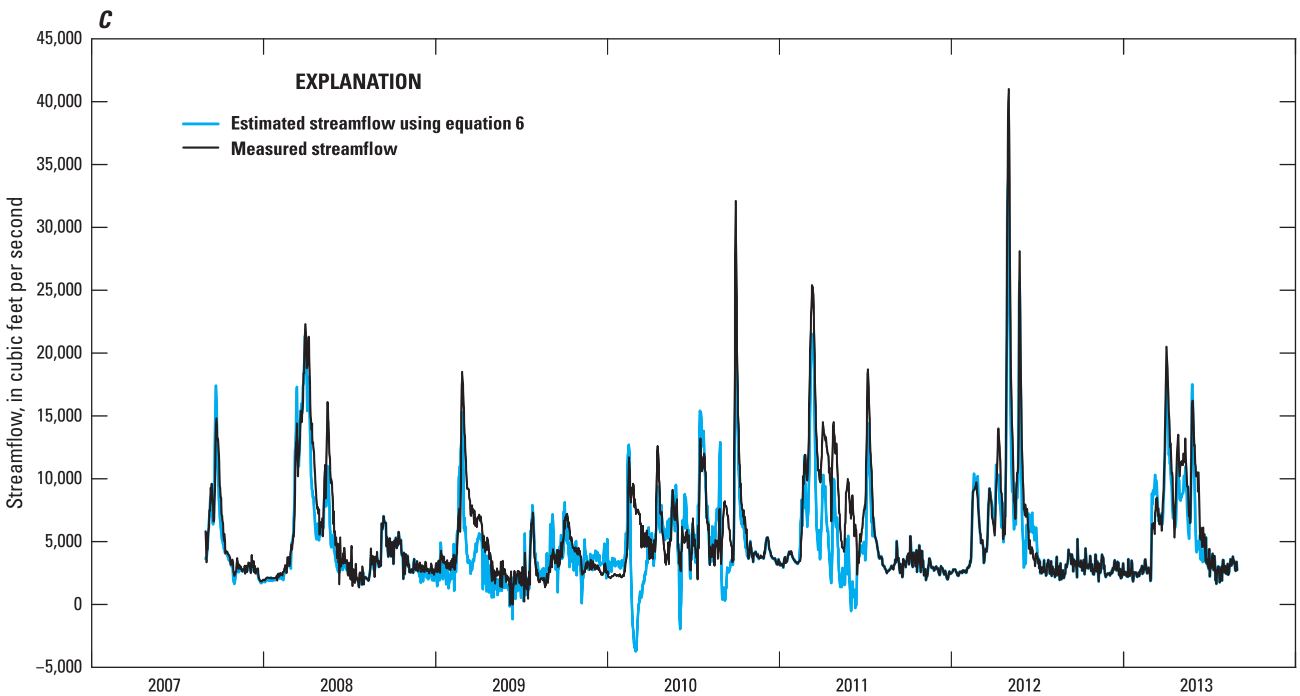


Figure 4. Time-series plots used to visually compare measured streamflows (water years 2008–13) for the St. Croix River at Prescott, Wisconsin (U.S. Geological Survey streamgage 05344490) to streamflows estimated using selected regression equations from table 3. *A*, equation 4; *B*, equation 5; *C*, equation 6; and *D*, equation 12.



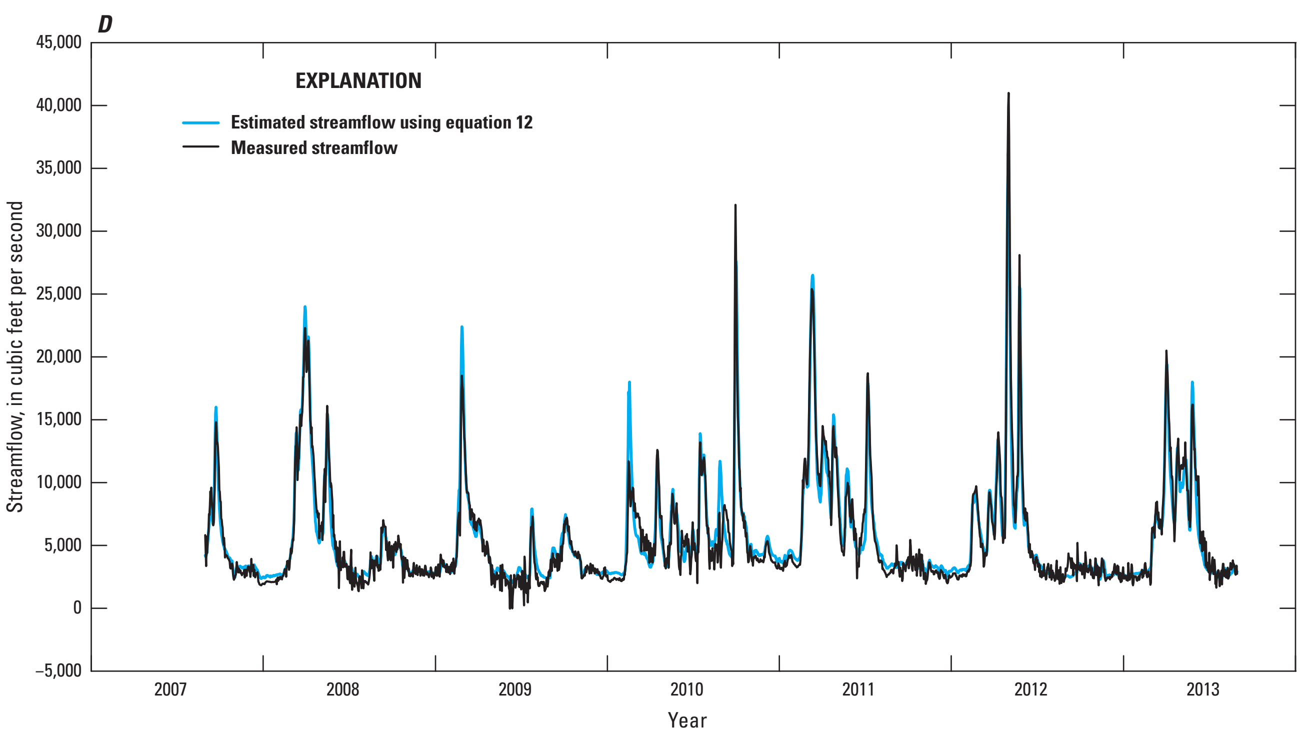


Figure 4. Time-series plots used to visually compare measured streamflows (water years 2008–13) for the St. Croix River at Prescott, Wisconsin (U.S. Geological Survey streamgage 05344490) to streamflows estimated using selected regression equations from table 3. *A*, equation 4; *B*, equation 5; *C*, equation 6; and *D*, equation 12.—Continued

compared to discrete streamflow measurements made in the span of about an hour, the discrete measurements may not capture all of the variability from precipitation, wind, or dam operation changes (on the St. Croix and Mississippi Rivers) that occurs in the course of a day at Prescott. The results of these comparisons are presented in table 7.

Percent-difference data in table 7 indicate that equation 12 is less accurate at estimating historical streamflows for Prescott than equation 9 is for estimating historical streamflows for Stillwater (table 6). However, estimation methods presented in this report indicate that Prescott is a hydrologically more complex location than Stillwater because of water storage in Lake St. Croix, contributions from two major upstream tributaries, and the confluence with the Mississippi River immediately downstream. No historical stage data are available for Prescott to help assess daily variations in streamflows, but daily estimated streamflows calculated using equation 12 overestimated discrete measurements for 10 of the 16 streamflow measurements. This tendency of overestimating measured values may indicate the backwater effects created by the confluence with the larger Mississippi River. In addition, the complexities of the site at Prescott likely result in larger within-day fluctuations in streamflow compared to Stillwater. Therefore, discrete streamflow measurements may not capture enough of the daily variability to be representative of daily mean streamflows. Furthermore, hydroacoustic instruments were not available for discrete streamflow measurements made during 1928–72 (table 7). Traditional methods of measuring streamflow may not be as accurate for sites like Prescott that have extremely variable flows affected by several factors, including upstream water storage in Lake St. Croix, backwater from the confluence with the Mississippi River, and wind direction (Rantz and others, 1982; Levesque and Oberg, 2012).

Table 7. Comparisons between estimated historical daily mean streamflows and discrete streamflow measurements made before the installation of an index-velocity streamgage at the St. Croix River at Prescott, Wisconsin.

[discrete *Qm*, discrete streamflow measured by U.S. Geological Survey hydrographers; daily *Qe*, daily mean streamflow estimated using equation 12 in the text and listed in table 1–1 for the corresponding date; ft^3/s, cubic feet per second; percent difference, the percent difference between the discrete *Qm* and the daily *Qe* using equation 8 from the text and listed in table 1–1]

Date of discrete streamflow measurement	Discrete *Qm* (ft^3/s)	Daily *Qe* (ft^3/s)	Percent difference
05/25/1928	5,620	4,890	-14.9
08/25/1928	2,640	3,430	+23.0
03/05/1930	5,980	3,550	-68.5
06/01/1933	3,970	5,290	+25.0
04/09/1934	5,290	5,640	+6.21
03/21/1935	3,880	5,480	+29.2
09/16/1953	5,200	3,680	-41.3
08/01/1957	4,620	5,400	+14.4
07/24/1962	2,960	3,230	+8.36
07/13/1965	2,590	4,070	+36.4
05/01/1967	5,070	6,150	+17.6
04/01/1969	2,920	5,960	+51.0
06/19/1972	3,960	3,780	-4.76
08/17/2000	3,300	3,820	+13.6
05/02/2006	6,030	5,870	-2.73
07/10/2007	2,470	2,290	-7.86

Comparisons of previous and revised estimates of historical streamflows at Prescott using equations 5 and 12, respectively (appendix 3), indicate noteable differences between the two streamflow estimation equations. Monthly and annual comparisons support the findings of regression analyses that previous regression equations generally underestimated low flows and overestimated high flows compared to the new regression equations (fig. 4). During the relatively dry, low-flow period of water years 1910–40, the previously estimated mean streamflow of 3,830 ft^3/s was 290 ft^3/s less than the revised estimated mean streamflow of 4,120 ft^3/s. In contrast, during the relatively wet, high-flow period of water years 1970–2000, the previously estimated mean streamflow of 5,910 ft^3/s was 30 ft^3/s greater than the revised mean streamflow of 5,880 ft^3/s. For the period of estimated historical streamflows (water years 1910–2007), the previously estimated mean streamflow of 5,140 ft^3/s was 90 ft^3/s less than the revised estimated mean streamflow of 5,230 ft^3/s, indicating that throughout the period of estimated historical streamflows, overestimates of peak flows were not high enough to balance the underestimates of low-flow conditions. However, the difference between previous and revised mean streamflows was less at Prescott than at Stillwater.

The differences between previous and revised estimates of historical streamflows may indicate the effects of groundwater contributions on streamflow at Prescott. Equation 5 used a simple scalar and generally underestimated base flows and overestimated peak flows in comparison to equation 12. The constant in equation 12 (y-intercept; 873) may represent a minimum base-flow value that was not accounted for in equation 5. In addition, the constant may better account for geologic differences between St. Croix Falls and Prescott. Downstream from St. Croix Falls, increased soil and bedrock permeability may allow the surrounding watershed to capture a greater portion of the water budget (Juckem, 2007), resulting in less peak runoff and greater low flows. Furthermore, the difference between previous and revised mean streamflow values may be lower at Prescott than Stillwater because of the effects of water storage in Lake St. Croix and because of larger groundwater contributions to Lake St. Croix from the Willow and Kinnickinnic Rivers. However, the hydrograph separation analyses needed to quantify low flows and peak flows at St. Croix Falls and Prescott are beyond the scope of this report.

Limitations of the Study

Several limitations should be considered when applying results obtained from analyses presented in this report. The regression analyses used to develop equations 9 and 12 for estimating revised historical streamflows at Stillwater and Prescott, respectively, violate some assumptions of regression analyses (Helsel and Hirsch, 2002). Because data from upstream streamgages are being used to estimate downstream streamflows, the explanatory and response variables are serially correlated. The variance of the residuals is not constant because the variance is dependent on the explanatory variables and time. In addition, the residuals are not normally distributed nor independent. Therefore, the analysis of variance (ANOVA) tables generated from the regression analyses are not presented in this report, and inferences should not be made about the variances of sample populations. The assumptions that are not violated in these analyses are that streamflows at Stillwater and Prescott are linearly related to the explanatory variables, and the data used to fit the models are representative of the data of interest. The non-violated assumptions allow the estimation of streamflows at Stillwater and Prescott given the explanatory variables used. However, the data presented in this report cannot be used to obtain a variance for estimated values, obtain an unbiased estimator of streamflow, or test hypotheses or estimate confidence intervals (Helsel and Hirsch, 2002). Because of the violated assumptions, NSE values (Nash and Sutcliffe, 1970) were used to assess the estimation powers of the hydrological equations used in this report.

Other limitations of this study are a result of the short period of record for the index-velocity streamgages that were used to evaluate previous methods of estimating streamflow and develop new equations to revise estimates of historical streamflows. First, equations 9 and 12 should only be considered valid for the range of measured streamflows for the explanatory variable used in the regression analyses (table 5). Second, the relation between explanatory and response variables has not been examined for periods of ice cover because no streamflow measurements have been made under ice cover at Stillwater (for safety reasons). Third, the limited periods of measured streamflow records from Stillwater and Prescott used to develop equations 9 and 12, respectively, may not capture all the variability in the streamflow records throughout the entire period of record for the explanatory variables. Finally, changes to dam operation (on the St. Croix and Mississippi Rivers), climate, or land and water use within the lower St. Croix River Basin may have altered the relation between the explanatory streamflows from select streamgages and streamflows at the index-velocity streamgages at Stillwater and Prescott.

Summary

A natural dam of glacial-era sediments at the confluence of the St. Croix and Mississippi Rivers forms Lake St. Croix, a riverine lake that comprises the lowest 25 miles of the St. Croix River. In 1993, concerns about future water quality in the St. Croix River (including Lake St. Croix) prompted several agencies and organizations to form the St. Croix Basin Water Resources Planning Team to coordinate research and monitoring efforts in the St. Croix Basin. Streamflow measurements for the St. Croix River at the inlet to (near Stillwater, Minnesota) and outlet from (near Prescott, Wisconsin) Lake St. Croix were needed to estimate nutrient loads. However, backwater effects from the Mississippi River prevented the use of traditional streamgages for collecting continuous streamflow data. Therefore, previous studies used streamgages upstream from the inlet to and outlet from Lake St. Croix and streamflow-routing equations to estimate streamflows needed for nutrient-load calculations. In the late 1990s, advances in velocity meters enabled researchers with the U.S. Geological Survey (USGS) to develop index-velocity streamgages to measure continuous streamflow in backwater conditions using continuously measured velocities at the streamgages. Index-velocity streamgages were installed on the St. Croix River at Prescott and Stillwater in 2007 and 2011, respectively.

This report was prepared by the U.S. Geological Survey in cooperation with the St. Croix Watershed Research Station. The purposes of this report are to (1) describe the development of regression equations to estimate historical streamflows of the St. Croix River at Stillwater and Prescott, (2) describe the evaluation of the accuracies of new and previous equations used to estimate historical streamflows, and (3) present revised estimates and evaluations of estimates of historical streamflows for the St. Croix River at Stillwater, Minn. (water years 1910–2011), and at Prescott, Wisc. (water years 1910–2007).

Continuous streamflow data from new index-velocity streamgages, long-term upstream streamgages, and tributary streamgages were used to evaluate previous equations used to estimate historical streamflows and develop new regression equations to improve estimates of historical streamflow at Stillwater and Prescott. The abilities of previous and new equations to accurately estimate (fit) measured streamflows were evaluated using Nash-Sutcliffe Efficiency (NSE) values. Comparisons of NSE values and time-series plots indicated that new regression equations estimated measured streamflows more reliably than previous estimation equations. The NSE values at Stillwater improved from 0.90 to 0.98, and the NSE values at Prescott improved from 0.77 to 0.94. New regression equations were used to compute revised estimates of historical streamflows for Stillwater and Prescott. Revised estimates of historical streamflows were compared to the most commonly used previous streamflow estimates for Stillwater and Prescott.

To assess the accuracies of revised estimates of historical streamflow, discrete streamflow measurements made before the installation of index-velocity streamgages were compared to revised estimates of historical daily streamflows. In general, estimates closely approximated measured discrete streamflows at Stillwater. Estimates of streamflows at Prescott were more variable when compared to measured discrete streamflows. Because daily streamflow estimates were compared to discrete streamflow measurements made in the span of about an hour, the discrete measurements may not capture all of the variability from precipitation, wind, or dam operation changes (on the St. Croix and Mississippi Rivers) that occurs in the course of a day at Prescott. In general, Prescott is a hydrologically more complex site than Stillwater because of water storage in Lake St. Croix directly upstream, the contributions of two major tributaries between Stillwater and Prescott, and the confluence with the Mississippi River directly downstream.

Additional comparisons were made between previous and revised estimates of historical streamflows at Stillwater and Prescott to evaluate changes in the estimated historical values. Monthly and annual streamflow comparisons indicated that previous streamflow estimates generally underestimated low flows and overestimated peak flows at Stillwater and Prescott when compared to revised estimates. For a 30-year dry period (water years 1910–40), the previously estimated mean streamflows were less than the revised estimated mean streamflows at Stillwater and Prescott. In contrast, for a 30-year wet period (water years 1970–2000), the previously estimated mean streamflow at Stillwater was slightly less than the revised estimated mean streamflow, whereas the previously estimated mean streamflow at Prescott was slightly greater than the revised estimated mean streamflow. For the entire periods of historical streamflow estimates, previously estimated mean streamflows were less than revised estimated mean streamflows at Stillwater and Prescott. Constant terms (*y*-intercepts) in the new regression equations may better represent low-flow conditions compared to previous equations, and differences in the relations between low flows and peak flows among regression equations may indicate changes in groundwater contributions resulting from increased soil and bedrock permeability downstream from the streamgage at St. Croix Falls from which the measured streamflow record is used as an explanatory variable in the regression equations for Stillwater and Prescott.

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Appendixes 1–3

Appendix 1. Summary of all Equations Listed in Text of the Report

Table 1–1. Summary of all the equations listed in the text of the report.

[All streamflow variables are in units of cubic feet per second. $Q_{Stillwater}$, estimated streamflow of the St. Croix River at Stillwater, Minnesota; Q_{SCF}, St. Croix River streamflow at St. Croix Falls, Wisconsin (U.S. Geological Survey streamgage 05340500); Q_{Apple}, streamflow of the Apple River near Somerset, Wisconsin (U.S. Geological Survey streamgage 05341500); $Q_{SCF\_lagged\_1\_day}$, streamflow of the St. Croix River at St. Croix Falls, Wisconsin (U.S. Geological Survey streamgage 05340500) lagged 1 day; Q_{Willow}, streamflow of the Willow River at Willow River State Park near Burkhardt, Wisconsin (U.S. Geological Survey streamgage 05341752); $Q_{Kinnickinnic}$, streamflow of the Kinnickinnic River near River Falls, Wisconsin (U.S. Geological Survey streamgage 05342000); $Q_{Prescott}$, estimated St. Croix River streamflow at Prescott, Wisconsin; $Q_{Miss\_Prescott}$; streamflow of the Mississippi River at Prescott, Wisconsin (U.S. Geological Survey streamgage 05344500); $Q_{St.Paul}$; streamflow of the Mississippi River at St. Paul, Minnesota (U.S. Geological Survey streamgage 05331000); Q_{MWTF}, discharge from the Metropolitan Wastewater Treastment Facility; ($Q_{St.Paul}$ + Q_{MWTF})lagged_1_day, sum of variables lagged by 1 day; *NSE*, Nash-Sutcliffe Efficiency; $Q_{m,i}$, measured daily mean streamflow at the index-velocity streamgage for each day i; *n*, upper bound of the summation; $Q_{e,i}$, estimated daily mean streamflow from the regression model for each day i; $\overline{Q}_m$, mean of all measured daily streamflows for the examined period of record; $Q_{Stillwater(WY2012)}$, estimated St. Croix River streamflow for water year 2012 at Stillwater, Minnesota; $Q_{Stillwater(WY2013)}$, estimated St. Croix River streamflow for water year 2013 at Stillwater, Minnesota; $Q_{SCF\_lagged\_3\_days}$, St. Croix River streamflow at St. Croix Falls, Wisconsin (U.S. Geological Survey streamgage 05340500) lagged 3 days.

Equation number	Equation	Reference(s)
1	$Q_{Stillwater} = Q_{SCF} + Q_{Apples}$	Kloiber, 2004; Triplett and others, 2009; LaFrancois and others, 2009.
2	$Q_{Stillwater} = 1.11 * Q_{SCF}$	Kloiber, 2004; Triplett and others, 2009; LaFrancois and others, 2009.
3	$Q_{Stillwater} = Q_{SCF\_lagged\_1\_day} + Q_{Apple\_lagged\_1\_day}$	Greg Mitton, U.S. Geological Survey, written commun, 2014.
4	$Q_{Prescott} = Q_{Stillwater} + Q_{Willow} + Q_{Kinnickinnic}$	Kloiber, 2004; Triplett and others, 2009; LaFrancois and others, 2009.
5	$Q_{Prescott} = 1.18 * Q_{SCF}$	Kloiber, 2004; Triplett and others, 2009; LaFrancois and others, 2009.
6	$Q_{Prescott} = Q_{Miss\_Prescott} - \{1.034[(Q_{St.Paul} + Q_{MWTF})_{lagged\_1\_day}]\}$	Greg Mitton, U.S. Geological Survey, written commun, 2014.
7	$NSE = 1 - \frac{\sum_{i=1}^{n} (Q_{m,i} - Q_{e,i})^2}{\sum_{i=1}^{n} (Q_{m,i} - \overline{Q}_m)^2}$	Nash and Sutcliffe, 1970.
8	$Percent\ Difference = \left[\frac{Q_e - Q_m}{Q_e}\right] * 100$	This report.
9	$Q_{Stillwater} = (0.968 * Q_{SCF\_lagged\_1\_day}) + 770$	This report.
10	$Q_{Stillwater(WY2012)} = (0.960 * Q_{SCF\_lagged\_1\_day}) + 907$	This report.
11	$Q_{Stillwater(WY2013)} = (0.971 * Q_{SCF\_lagged\_1\_day}) + 680$	This report.
12	$Q_{Prescott} = (0.585 * Q_{SCF\_lagged\_1\_day}) + (0.415 * Q_{SCF\_lagged\_3\_day}) + 873$	This report.

Appendix 2. Previous and Revised Estimates of Historical Daily Streamflows for the St. Croix River at Stillwater, Minnesota

Previous and revised estimates of historical daily streamflows for the St. Croix River at Stillwater, Minnesota are presented as a Microsoft Excel® workbook (*http://pubs.usgs.gov/sir/2014/5239/downloads/Appendix2.xlsx*). All streamflow values listed in appendix 2 are in cubic feet per second. Previous estimates of streamflow were calculated using equation 2 from the text and listed in 1–1, and revised estimates are calculated using equation 9 from the text and listed in table 1–1. The first tab of the Excel worksheet contains estimated streamflow values for the period of record (1910–2011 water years), and other tabs contain daily streamflow values for an individual water year, organized monthly similar to the way daily streamflow values are published in USGS Annual Data Reports (*http://wdr.water.usgs.gov/*).

Appendix 3. Previous and Revised Estimates of Historical Daily Streamflows for the St. Croix River at Prescott, Wisconsin

Previous and revised estimates of historical daily streamflows for the St. Croix River at Prescott, Wisconsin, are presented as a Microsoft Excel® workbook (*http://pubs.usgs.gov/sir/2014/5239/downloads/Appendix3.xlsx*). All streamflow values listed in appendix 3 are in cubic feet per second. Previous estimates of streamflow were calculated using equation 5 from the text and listed in table 1–1, and revised estimates are calculated using equation 12 from the text and listed in table 1–1. The first tab of the Excel worksheet contains estimated streamflow values for the period of record (1910–2007 water years), and other tabs contain daily streamflow values for an individual water year, organized monthly similar to the way daily streamflow values are published in USGS Annual Data Reports (*http://wdr.water.usgs.gov/*).

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