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Elastic-Waveform Inversion with Compressive Sensing for Sparse Seismic Data

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ABSTRACT

Accurate velocity models of compressional- and shear-waves are essential for geothermal reservoir characterization and microseismic imaging. Elastic-waveform inversion of multi-component seismic data can provide high-resolution inversion results of subsurface geophysical properties. However, the method requires seismic data acquired using dense source and receiver arrays. In practice, seismic sources and/or geophones are often sparsely distributed on the surface and/or in a borehole, such as 3D vertical seismic profiling (VSP) surveys. We develop a novel elastic-waveform inversion method with compressive sensing for inversion of sparse seismic data. We employ an alternating-minimization algorithm to solve the optimization problem of our new waveform inversion method. We validate our new method using synthetic VSP data for a geophysical model built using geologic features found at the Raft River enhanced-geothermal-system (EGS) field. We apply our method to synthetic VSP data with a sparse source array and compare the results with those obtained with a dense source array. Our numerical results demonstrate that the velocity models produced with our new method using a sparse source array are almost as accurate as those obtained using a dense source array.

1. INTRODUCTION

Elastic-waveform inversion (EWI) has potential to accurately estimate subsurface geophysical properties (Tarantola, 1984; Mora, 1987; Pratt et al., 1998; Sirgue and Pratt, 2004). EWI usually relies on a dense data coverage. In practice, seismic sources and/or receivers are often sparse. Candes et al. (2006) and Donoho (2006) demonstrated that signal information can be reconstructed from fewer linear measurements than previously thought possible. This technique is termed the compressive sensing. The technique makes use of the sparsity of the inversion in a certain transformed domain. Commonly used transformed domains include gradient domain (Chartrand, 2012; Sidky et al., 2013) or wavelet domain (Candes et al., 2006).

We develop an elastic-waveform inversion method with L_p norm based compressive sensing technique to preserve the accuracy of velocity estimation with significantly reduced data coverage. We employ the sparsity in the gradient domain because of its success applications in other applications such as medical imaging and image analysis. To promote the sparsity of the inversion, L_1 norm related minimization is usually used. Chartrand (2012) shows that a L_p norm minimization yields a more accurate inversion than the conventional L_1 norm mimization. This also changes the conventional compressive sensing from a convex optimization problem to a nonconvex optimization. To solve the nonconvex minimization problem, we use an alternating direction method of minimization (ADMM), which yields a very efficient computational algorithm.

We use synthetic VSP data for an elastic model to validate the capability of our new elastic-waveform inversion with a compressive sensing technique for estimating both the compressional and shear velocities using sparse seismic data. This preliminary elastic model is built using geologic features found at the Raft River geothermal field (Ayling and Moore, 2013). Our numerical example demonstrates that our new elastic-waveform inversion method preserves the accuracy of compressional and shear velocity inversion using only a fraction of seismic data needed for conventional elastic-waveform inversion.

2. ELASTIC-WAVEFORM INVERSION WITH COMPRESSIVE SENSING

The forward modeling of elastic-wave propagation can be written as

$$\mathbf{p} = f(\mathbf{K}, \rho, \mathrm{s}), \tag{1}$$

where $\mathbf{p}$ is the elastic-wavefield, f is the propagation operator, ρ is the density, $\mathbf{K}$ is the elastic moduli, and s is the source term. Numerical techniques, such as finite difference and spectral element methods, can be used to solve for forward problem. Let $\mathbf{m}$ be the model parameters, Eq. (1) becomes

$$\mathbf{p} = f(\mathbf{m}). \tag{2}$$

Elastic-waveform inversion is to solve the minimization problem

$$E(\mathbf{m}) = \min_{\mathbf{m}} \left\{ \| \mathbf{d} - f(\mathbf{m}) \|_2^2 \right\}, \tag{3}$$

where $E(\mathbf{m})$ is the misfit function, $\|\cdot\|_2$ stands the L_2 norm, and $\mathbf{d}$ represents recorded elastic-waveforms. The resulting model $\mathbf{m}$ minimizes square difference between recorded and synthetic waveforms.

We employ the compressive sensing technique (Chartrand, 2012; Sidky et al., 2013) in elastic-waveform inversion, and the cost function is given by

$$E(\mathbf{m}) = \min_{\mathbf{m}}\{\|\mathbf{d} - f(\mathbf{m})\|_2^2 + \lambda\|\nabla\mathbf{m}\|_p^p\}, \tag{4}$$

where $0 \le p \le 1$, λ is a positive regularization parameter, and the compressive sensing term $\|\nabla\mathbf{m}\|_p^p$ for a 2D model is defined as the L_p norm given by

$$\|\nabla\mathbf{m}\|_p^p = \sum_{i,j}(|(\nabla_x\mathbf{m})_{i,j}| + |(\nabla_z\mathbf{m})_{i,j}|)^p, \tag{5}$$

where $(\nabla_x\mathbf{m})_{i,j} = \mathbf{m}_{i+1,j} - \mathbf{m}_{i,j}$ and $(\nabla_z\mathbf{m})_{i,j} = \mathbf{m}_{i,j+1} - \mathbf{m}_{i,j}$.

We select $p = 1/2$ according to Chartrand (2012). Similar to Chartrand (2012), we employ an alternating direction method of minimization to solve the Eq. (4).

3. NUMERICAL RESULTS

We use synthetic VSP data for an elastic model shown in Fig. 1 to demonstrate the capability of our new elastic-waveform inversion method with the compressive sensing technique for velocity inversion using sparse seismic data. This preliminary elastic model is constructed using geologic features found at the Raft River geothermal field (Ayling and Moore, 2013). It contains two major fault zones that have been identified on the west side of the valley: the bridge fault zone and the horse wells fault zone. The model also contains a vertical narrow structure. 170 sources are evenly distributed with a spatial interval of 14 m at the top surface of the model and 162 receivers are placed along a vertical well across the center of the vertical narrow structure. The receivers are located between 0.5 km and 1.8 km in depth. A Ricker wavelet with a center frequency of 40 Hz is used as the source function.

We smooth the original velocity models in Fig. 1 by averaging the slownesses within two wavelengths at the center frequency of the source wavelet, resulting in the smoothed models in Fig. 2. We use these smoothed models as the starting models for EWI inversions.

Figure 3 shows the result of EWI inversion using all seismic data, and Fig. 4 displays the result of EWI with the compressive sensing technique using only one tenth of the original data (or one tenth of the total seismic sources). EWI with the compressive sensing technique yields compressional and shear velocity models similar to those obtained using EWI.

For comparison of the updated velocity values estimated using the two EWI inversions, we show the velocity updates from the starting models in Figs. 5 and 6. Even though only one tenth of the data coverage are used in the EWI with the compressive sensing technique, the updates of both compressional and shear velocities are similar to those obtained with EWI using all data.

4. CONCLUSIONS

We have developed a novel elastic-waveform inversion method with the compressive sensing technique for velocity inversion of sparse seismic data. The method employs an L_p norm in the regularization term. We have validated the capability of our new elastic-waveform inversion method for velocity inversion using only a fraction of seismic data acquired using a dense seismic source array. Our elastic-waveform inversion results of synthetic VSP data for a Raft River geothermal velocity model demonstrate that our new method can preserve the accuracy of elastic velocity inversion with sparse seismic data.

5. ACKNOWLEDGMENTS

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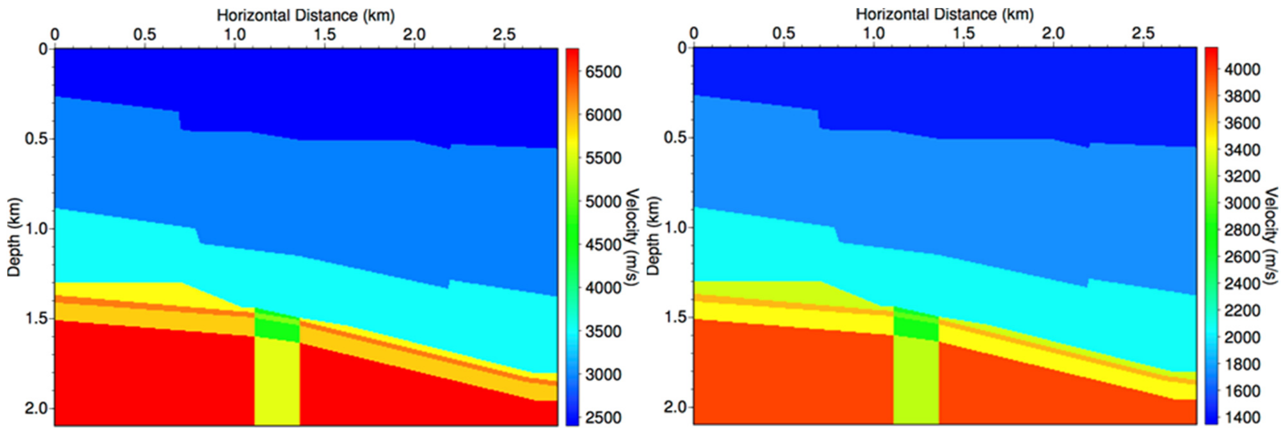
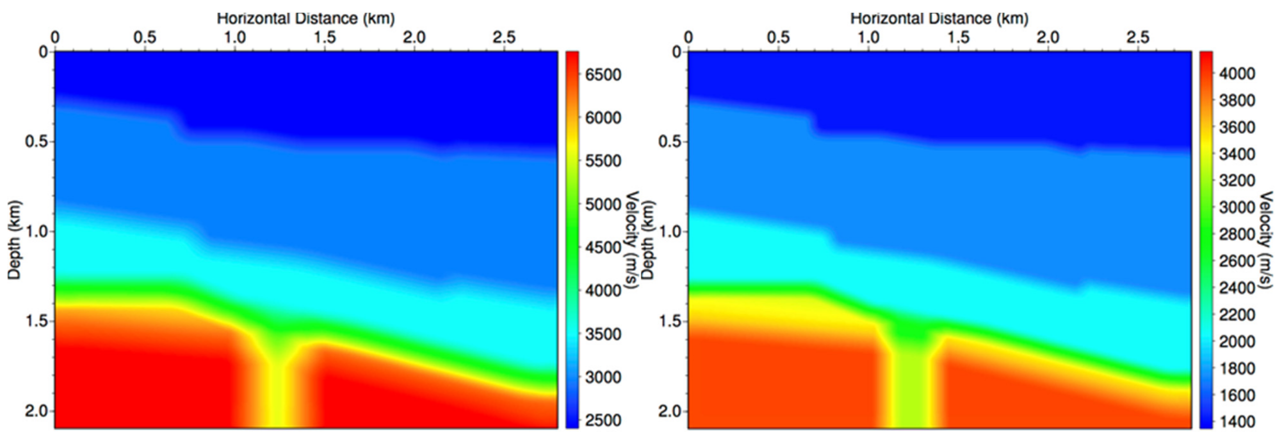


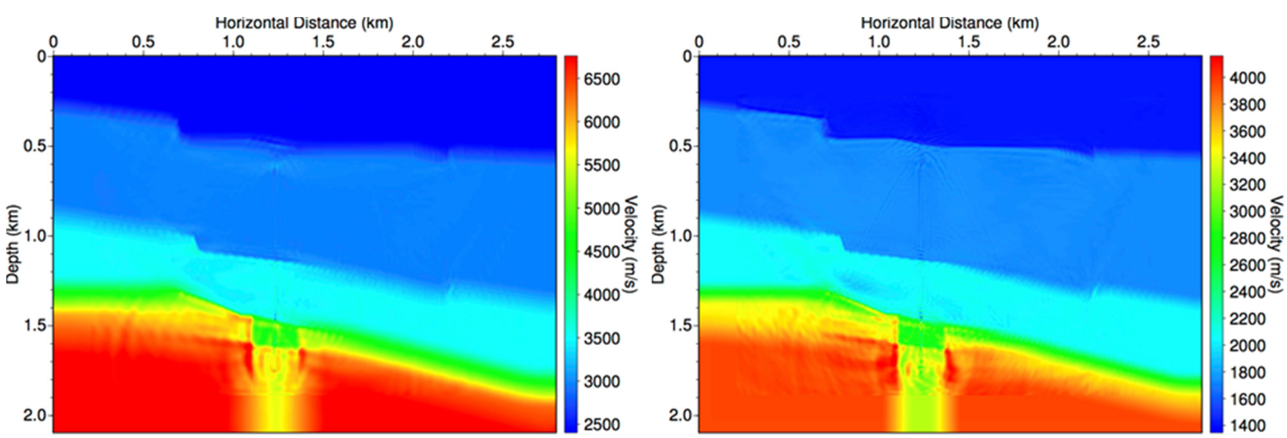
(a) (b)

Figure 1: True compressional (a) and shear (b) velocity models built based on the geologic features found at the Raft River geothermal field (Modified from Ayling and Moore, 2013).



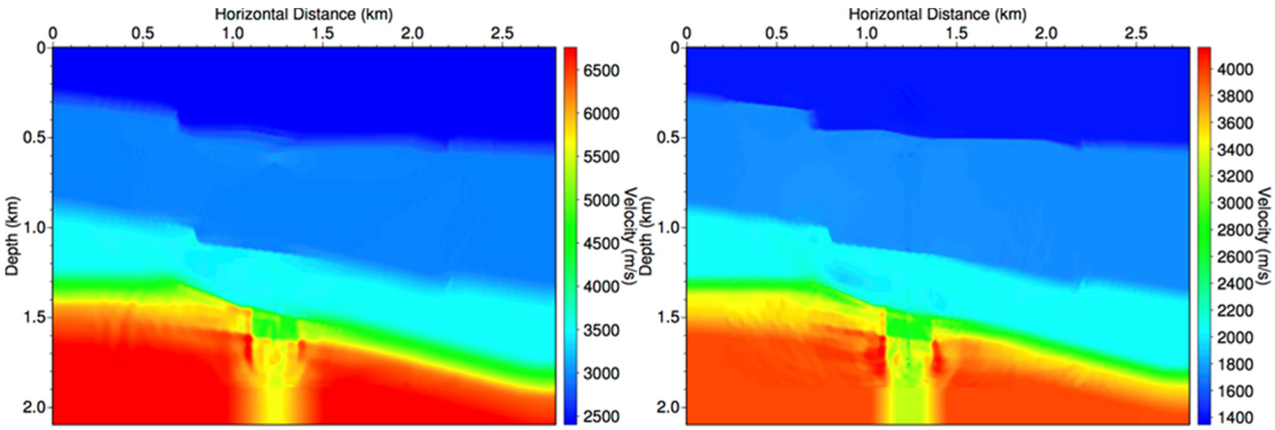
(a) (b)

Figure 2: Smoothed compressional (a) and shear (b) velocity models used as the starting models for elastic-waveform inversion.



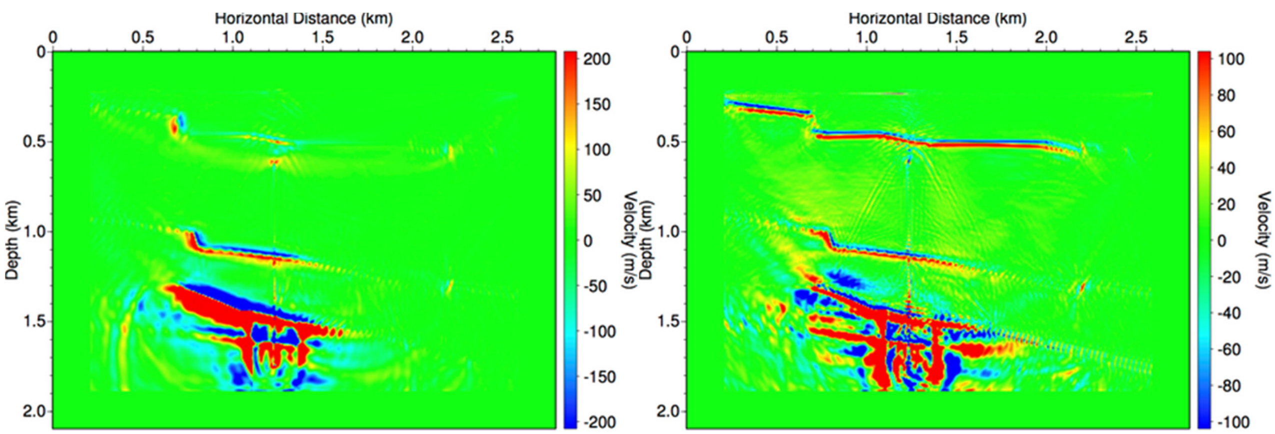
(a) (b)

Figure 3: Inverted compressional (a) shear (b) velocity models obtained with EWI using all seismic data.



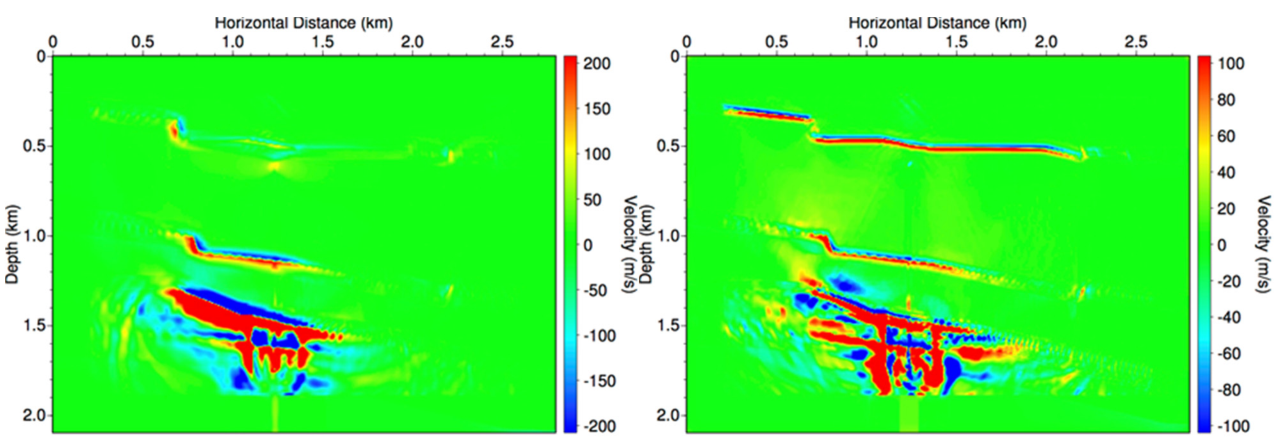
(a) (b)

Figure 4: Inverted compressional (a) shear (b) velocity models obtained using our EWI method with the compressive sensing technique using only one tenth of the original seismic data.



(a) (b)

Figure 5: Compressional (a) and shear (b) velocity model updates obtained with EWI using all seismic data.



(a) (b)

Figure 6: Compressional (a) and shear (b) velocity model updates obtained using our EWI method with the compressive sensing technique using only one tenth of the original seismic data.

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